Generalized Lifting for Sparse Image Representation and Coding

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CITEDI Research Center

Outline

- ✓ Introduction
  - Motivation
  - Previous work
- ✓ Lifting structures
  - Classical lifting
  - Generalized lifting
- ✓ Coding scheme
- ✓ Experimental results
  - Energy and entropy
  - Rate-distortion
  - Perceptual quality
- ✓ Conclusions

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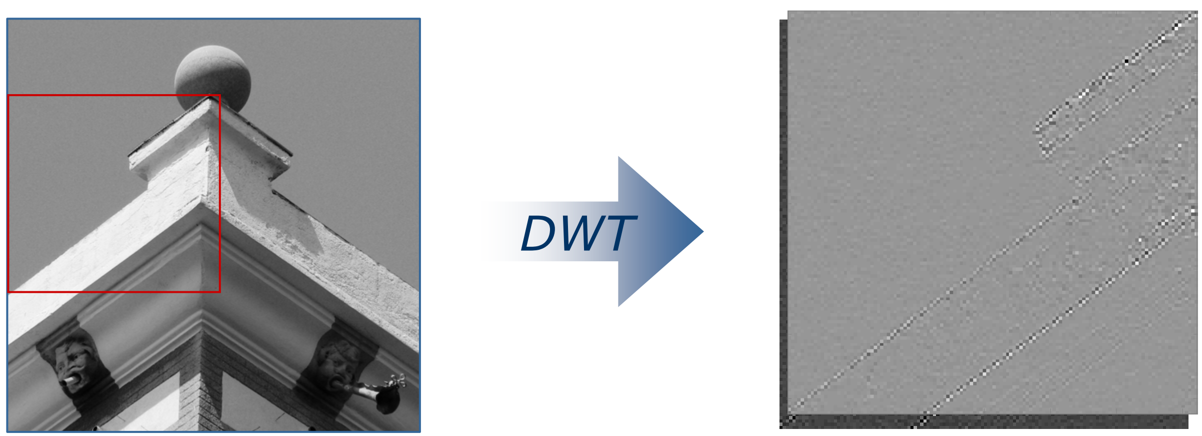
Introduction: Motivation

✓ Limitations of wavelets in image representation

- Excellent for decorrelation of impulsional-like events
- Limitations on images because wavelets are not able to fully decorrelate edges



LH subband zoom

Introduction: Motivation

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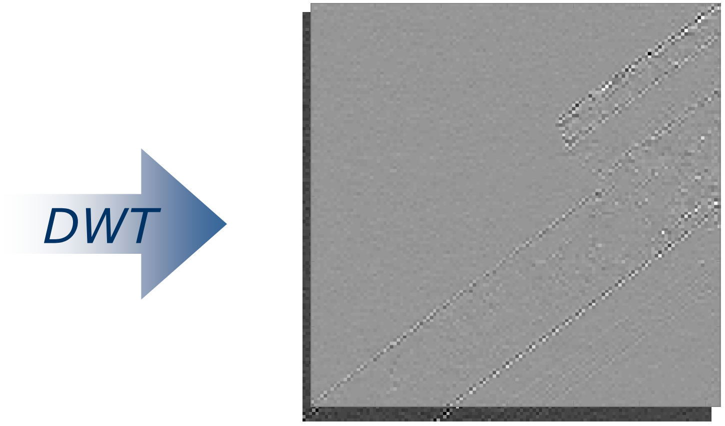

LH subband zoom

Need of additional decorrelation for coding applications

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Introduction: Previous work

Main approaches

- Go beyond wavelets trying to overcome their limitations
- Edges become 1-dimensional localizable singularities
- Increased sparsity of the coefficients

Frequency-domain methods

Curvelets
[Candès,Donoho 1999; Candès 2006]

Contourlets
[Do,Vetterli 2005]

Spatial-domain methods

Bandelets
[LePennec,Mallat 2005; Peyré,Mallat 2005]

Adaptive directional lifting
[Chappelier *et al* 2006; Ding *et al* 2007; Heijmans *et al* 2005; Mehrseresht,Taubman 2006; Piella *et al* 2002,2006; Wang *et al* 2006; Zhang *et al* 2005]

Generalized lifting
May be highly non-linear, preserving perfect reconstruction

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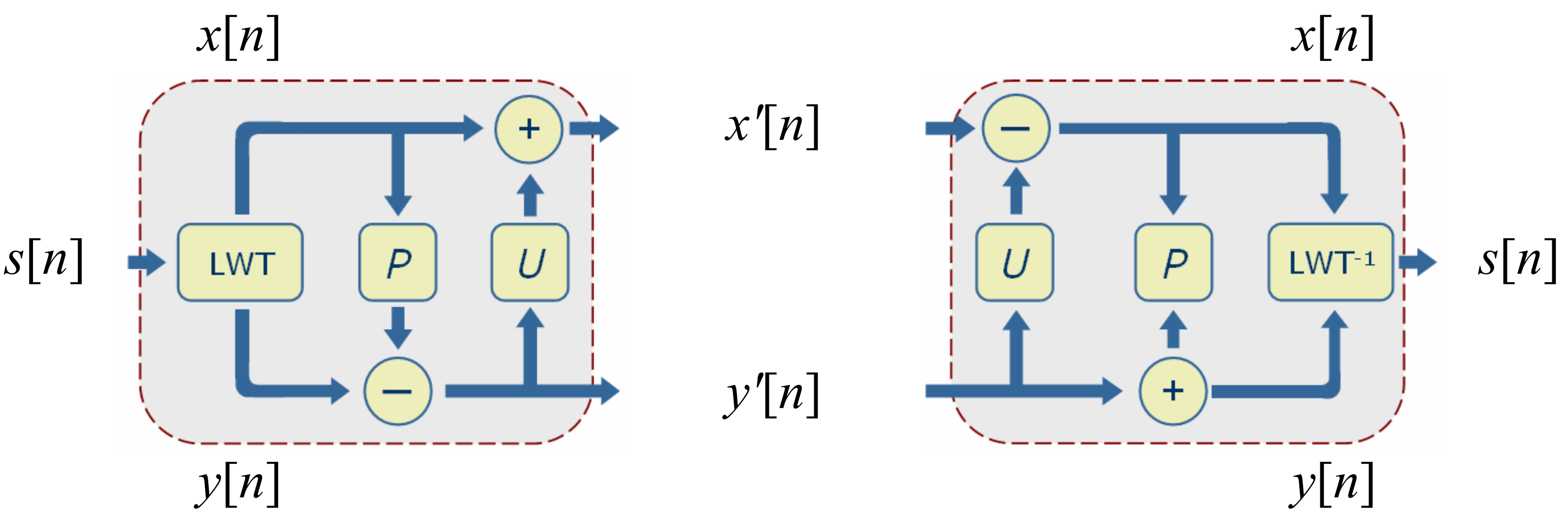
Lifting structures: Classical lifting

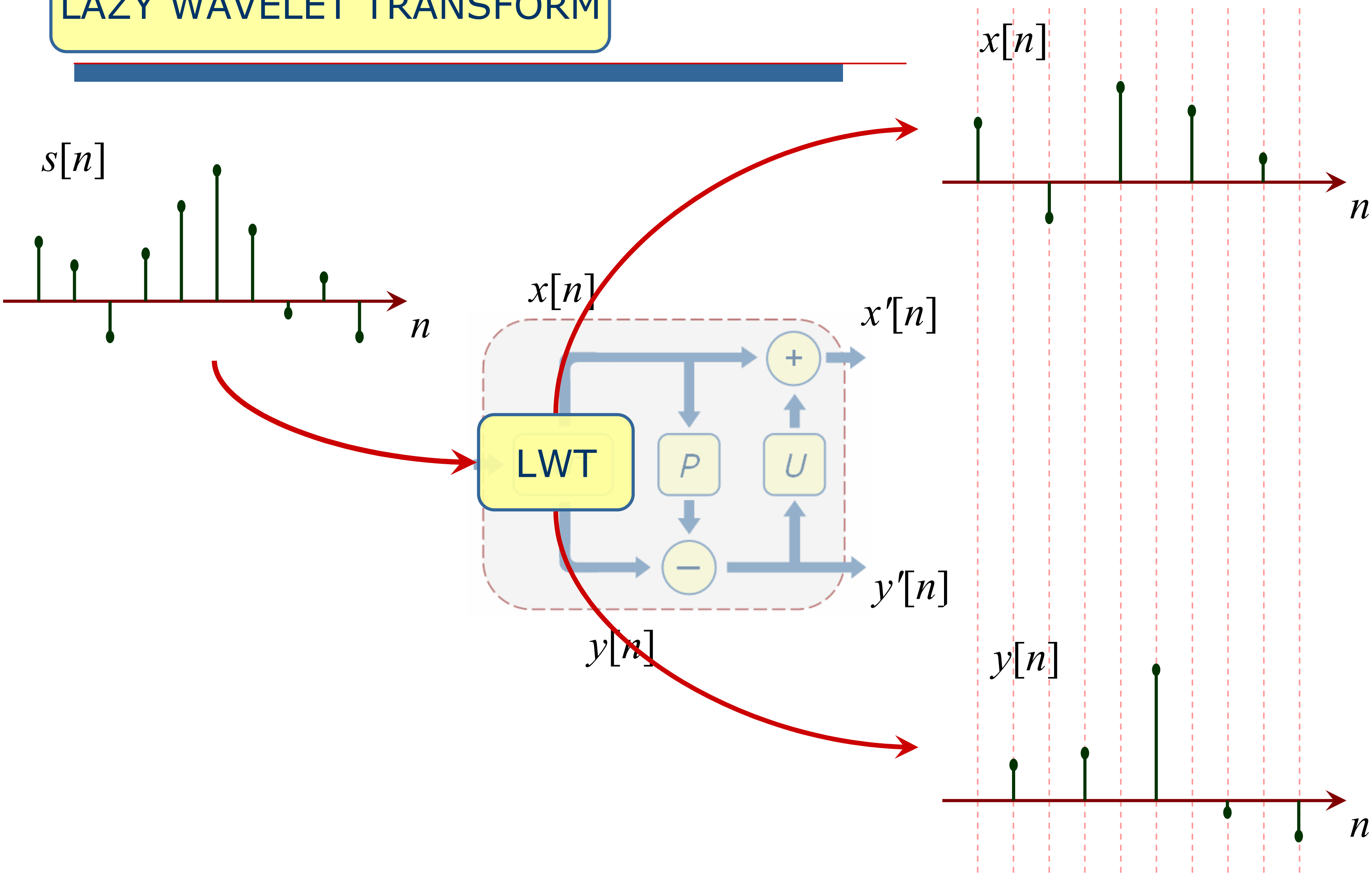
Approximation signal: $x'[n] = x[n] + \mathrm{U}(y'[n])$



Detail signal: $y'[n] = y[n] - \mathrm{P}(x[n])$

[Sweldens 1996]

LAZY WAVELET TRANSFORM



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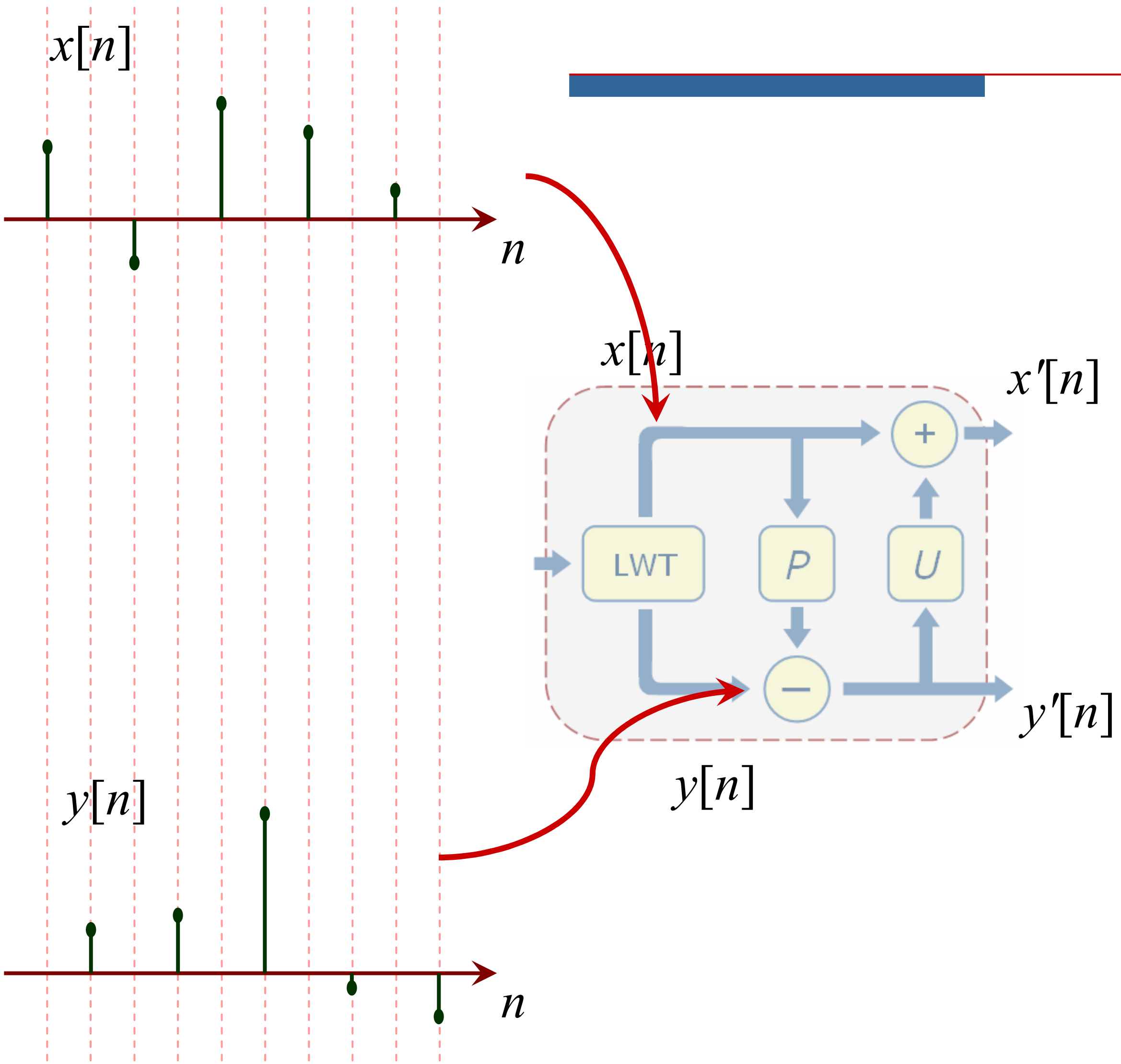

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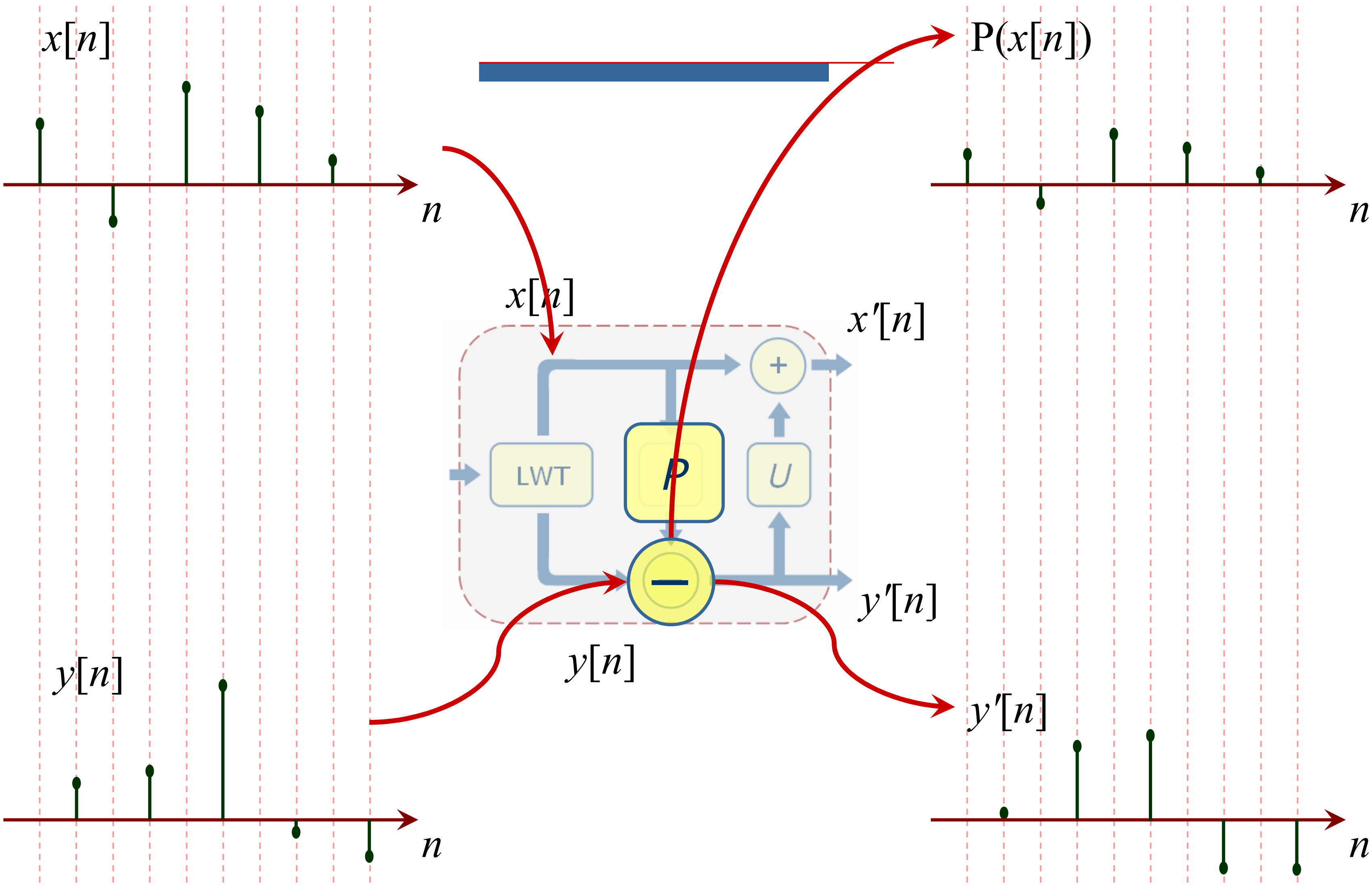

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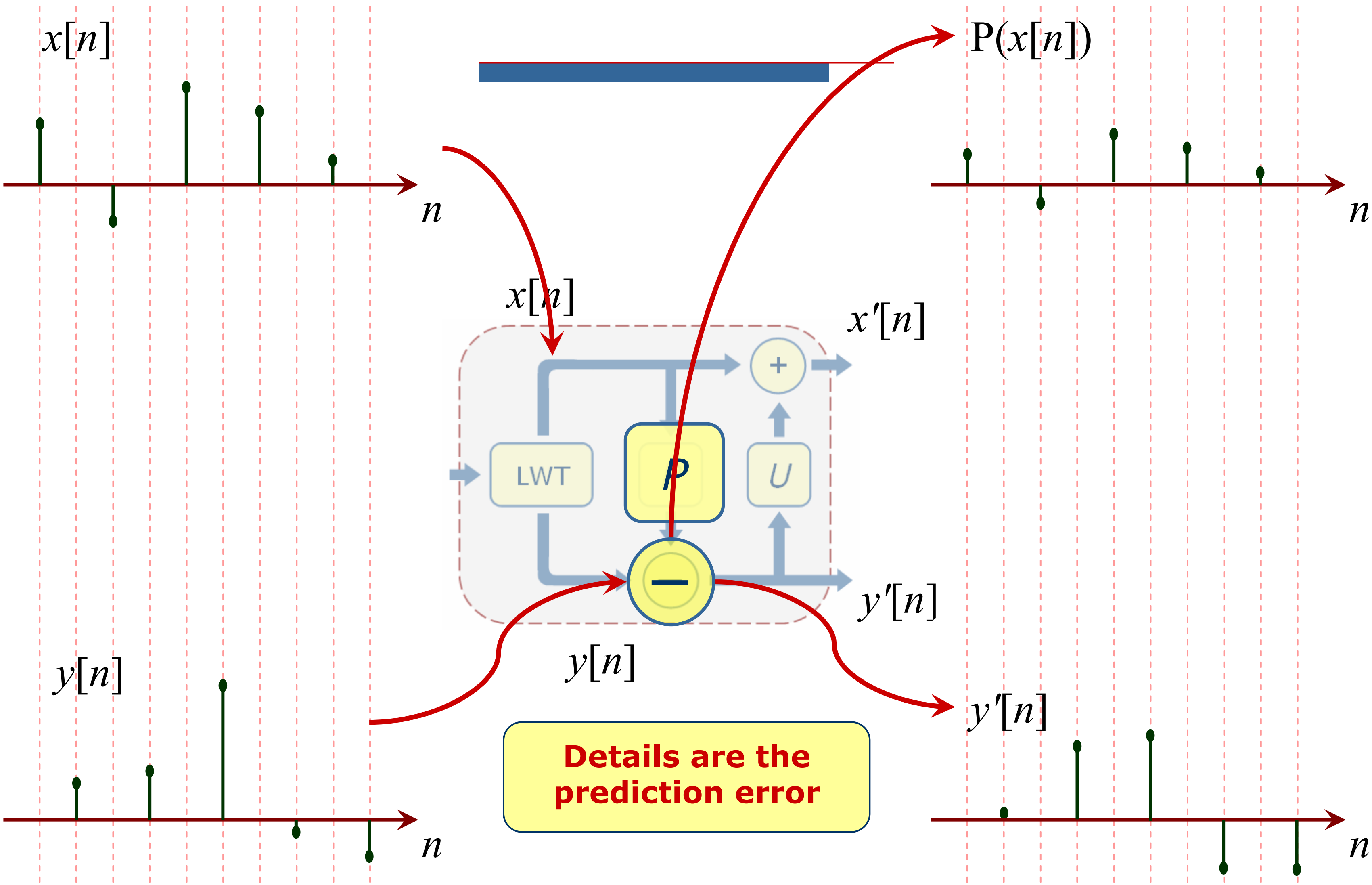

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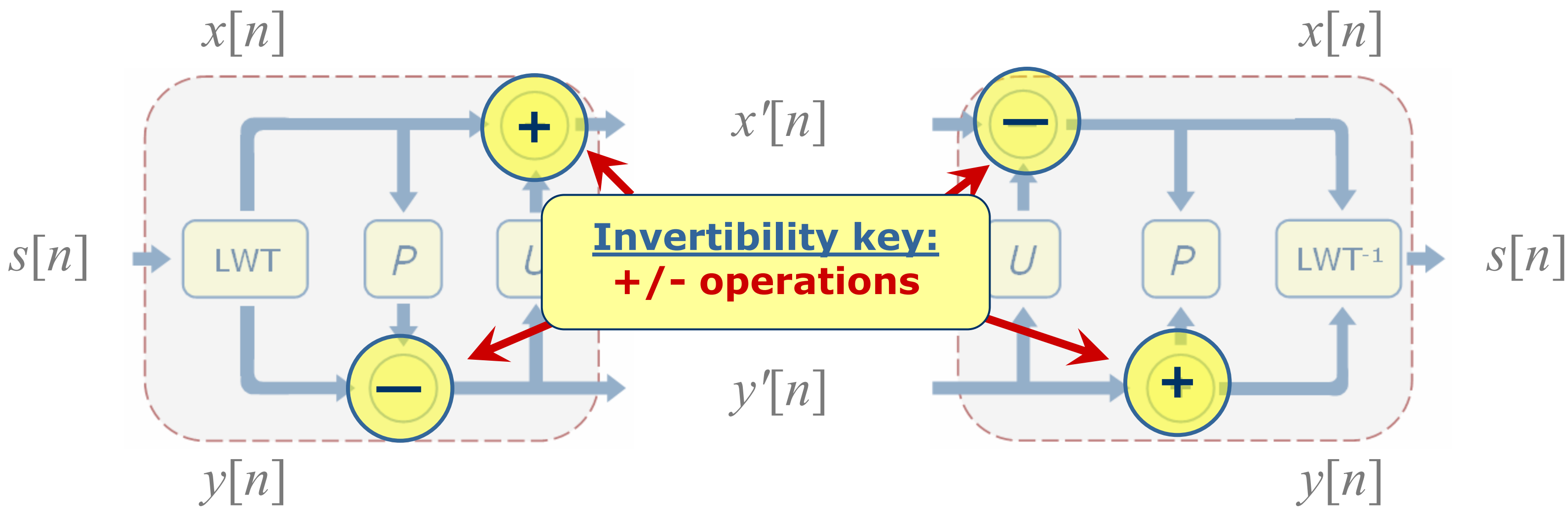

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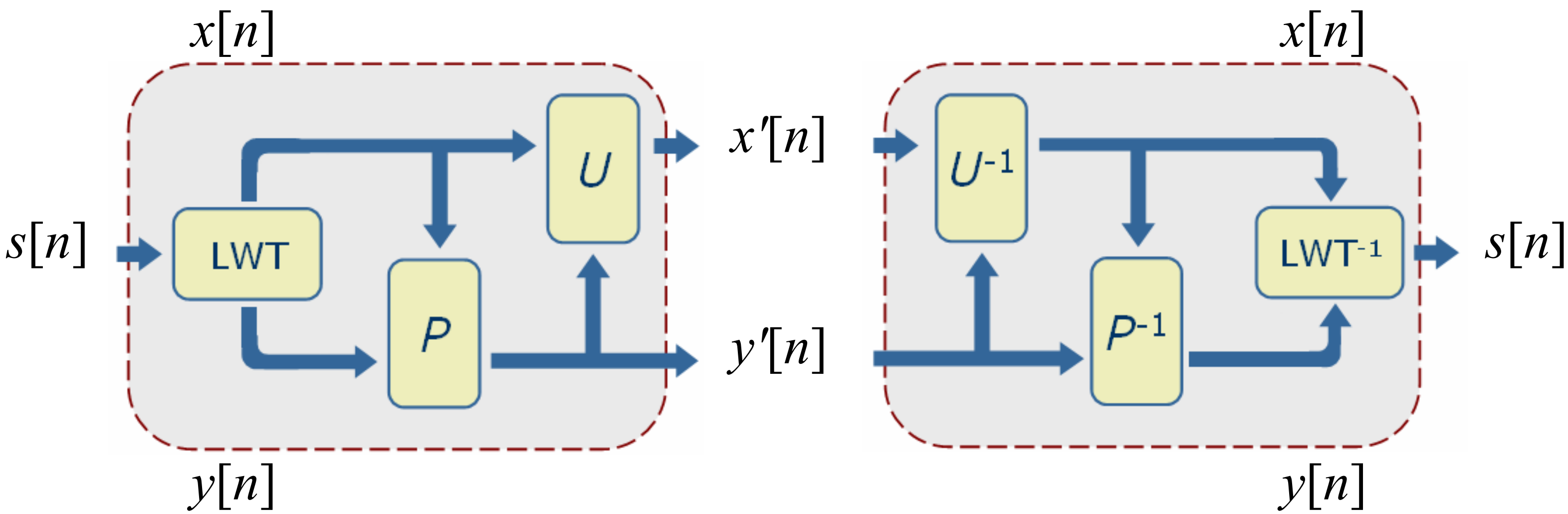
Lifting structures: Generalized lifting

Approximation signal: $x'[n] = \mathrm{U}(x[n], y'[n])$



Detail signal: $y'[n] = \mathrm{P}(y[n], x[n-i])|_{i \in C}$

[Solé,Salembier 2004]

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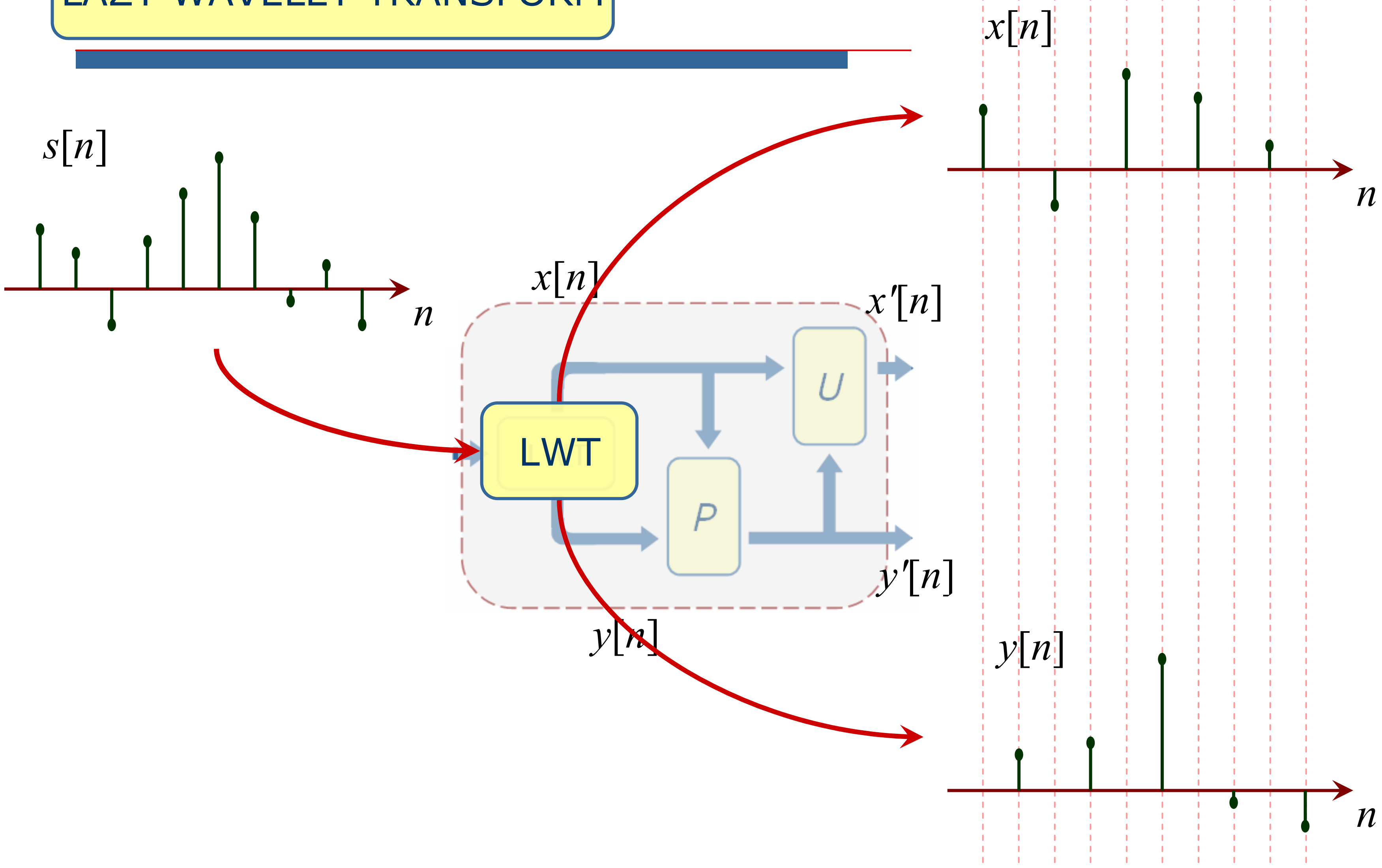

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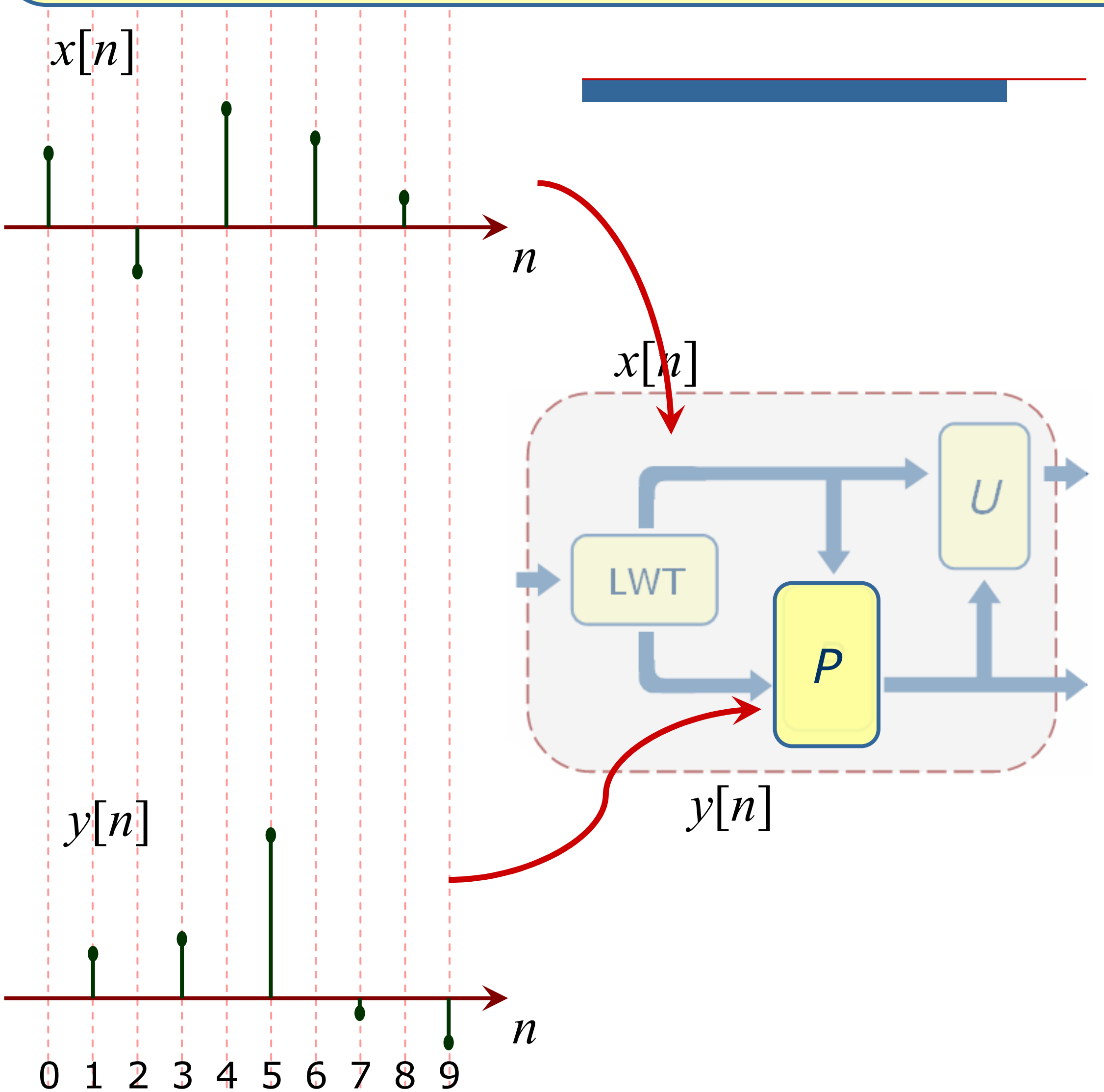

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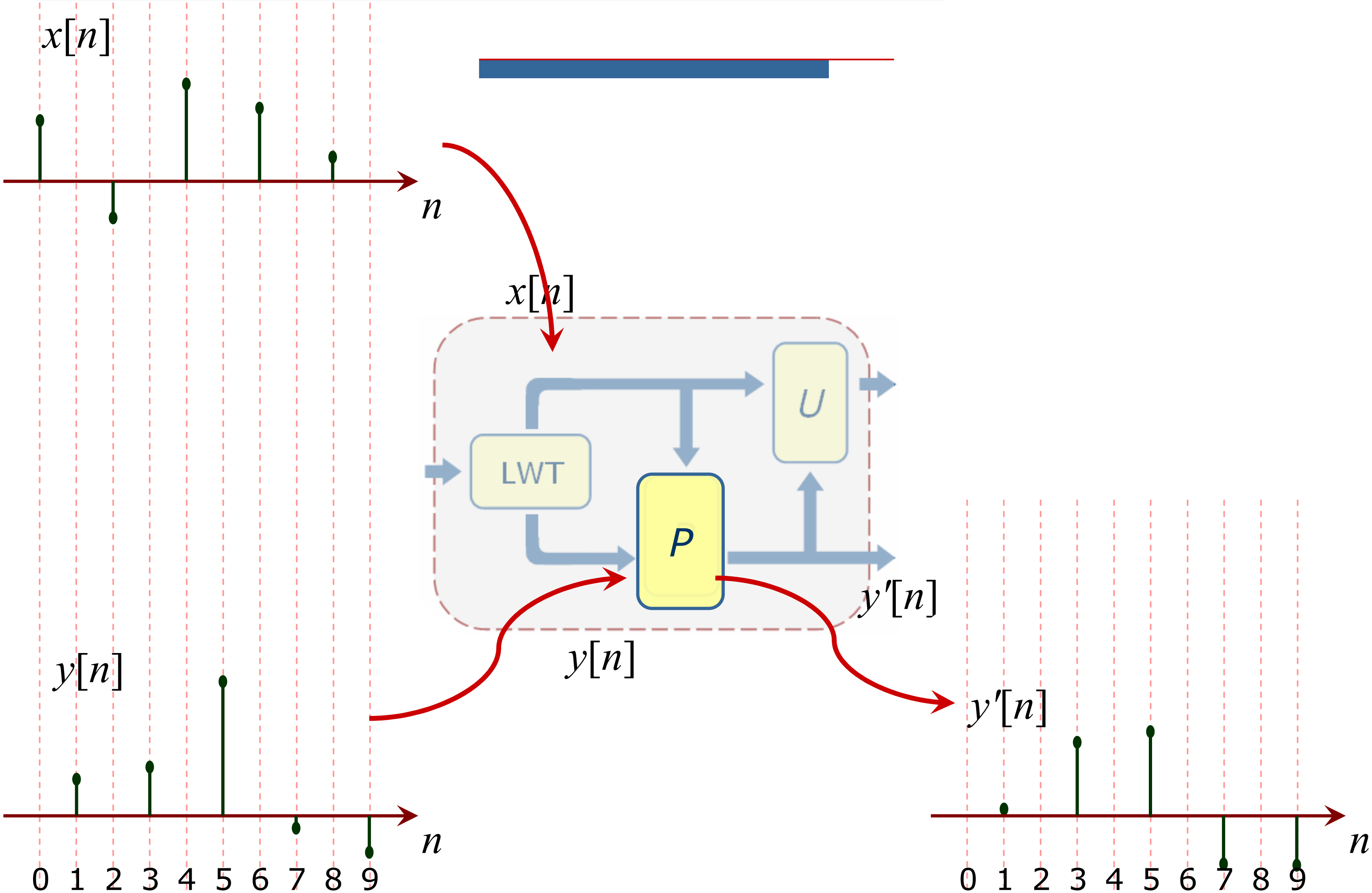

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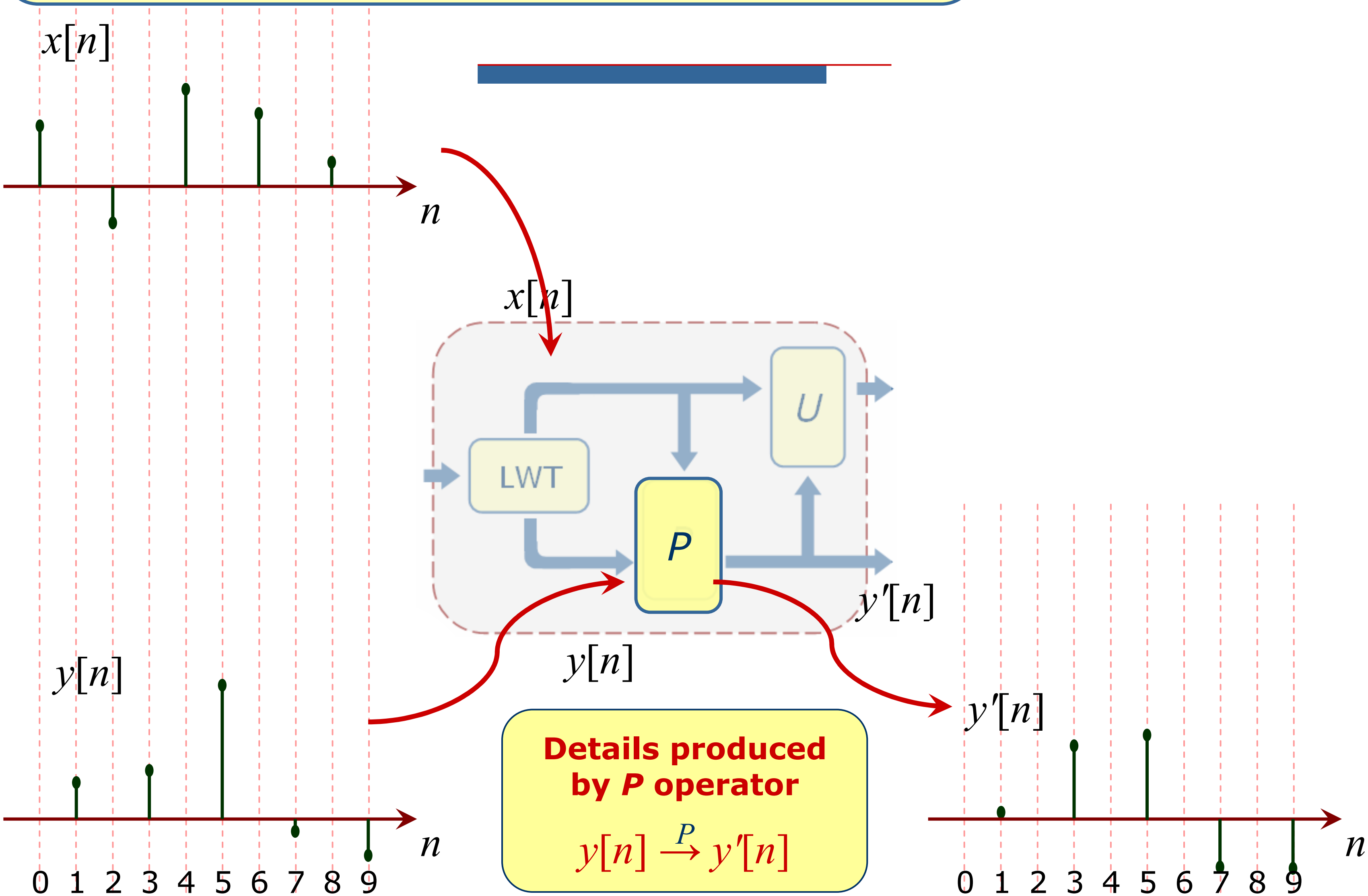

Details produced by *P* operator

$y[n] \xrightarrow{P} y'[n]$

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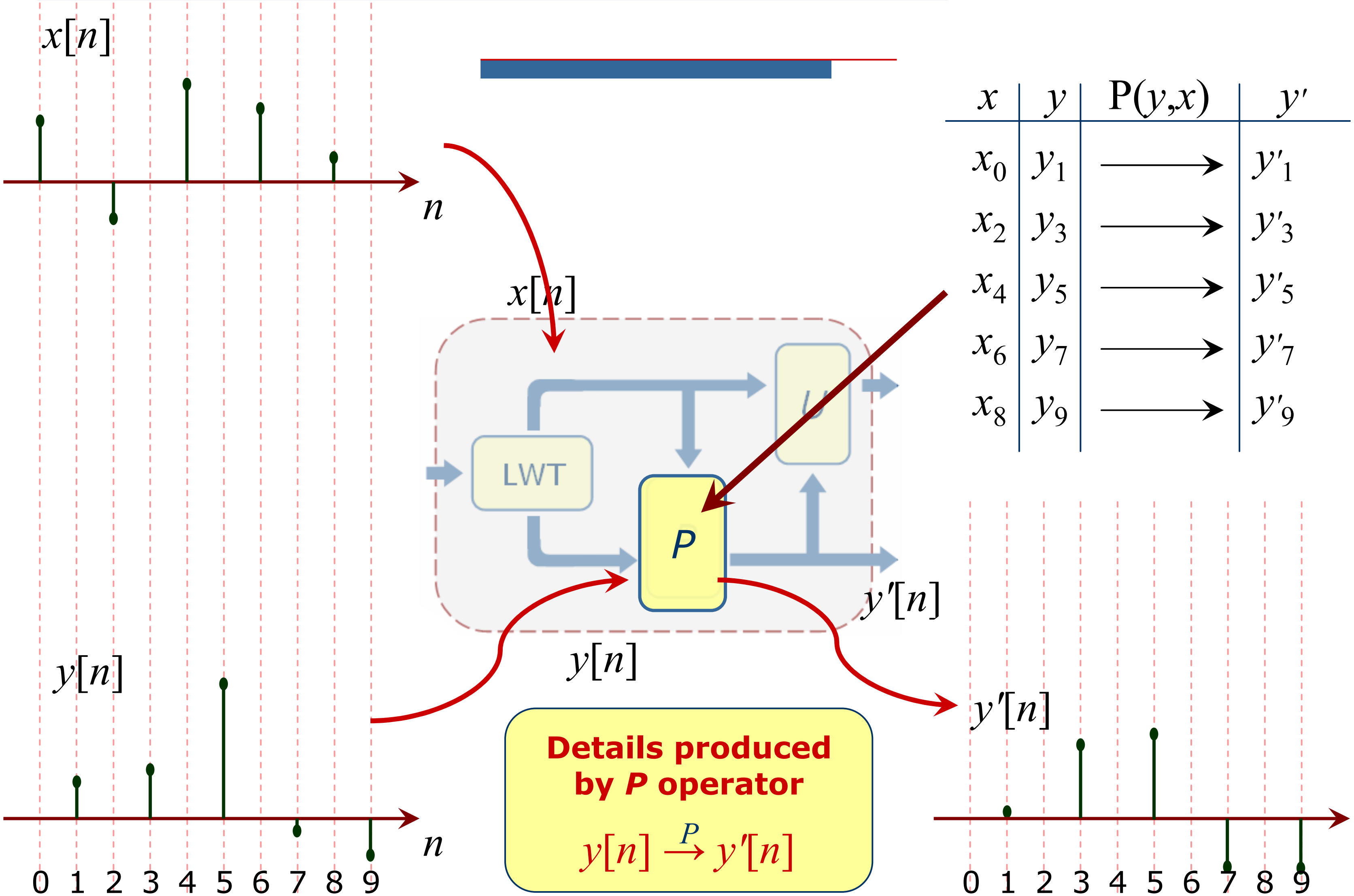
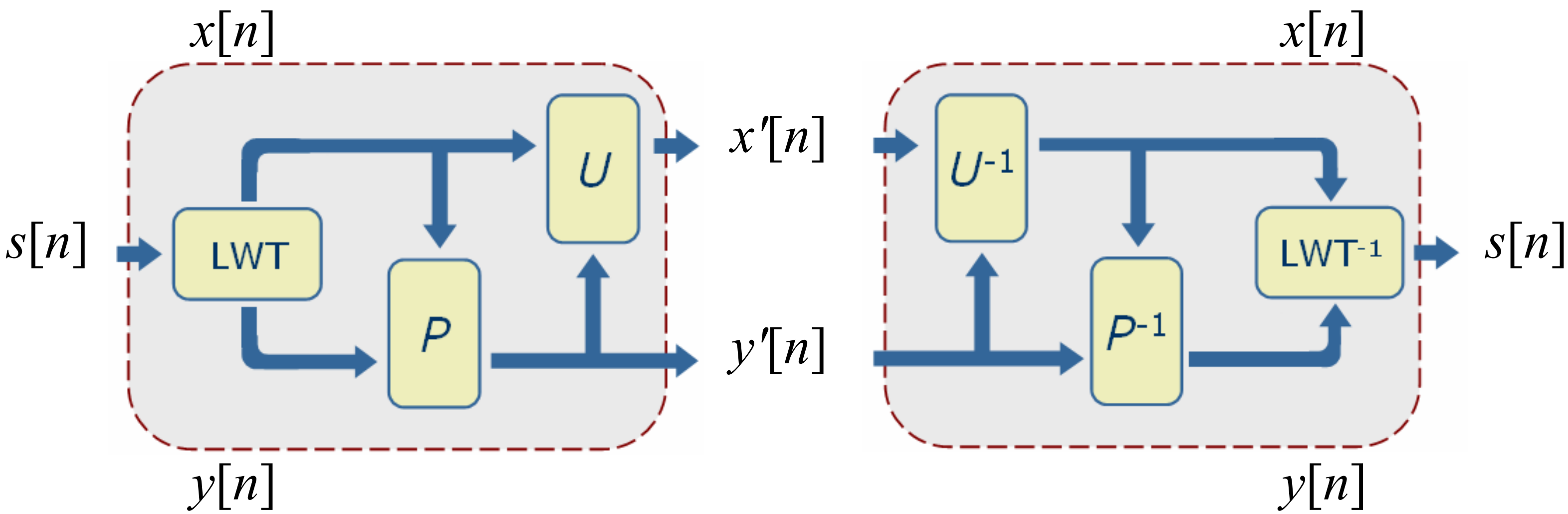


x	y	$\mathrm{P}(y,x)$	y'
x_0	y_1	$\longrightarrow$	y'_1
x_2	y_3	$\longrightarrow$	y'_3
x_4	y_5	$\longrightarrow$	y'_5
x_6	y_7	$\longrightarrow$	y'_7
x_8	y_9	$\longrightarrow$	y'_9

Lifting structures: Generalized lifting

Approximation signal: $x'[n] = U(x[n], y'[n])$



De

BIJECTIVITY OF *P*, *U* IS MANDATORY
TO ACHIEVE PERFECT RECONSTRUCTION

[Solé,Salembier 2004]

Lifting structures: Generalized lifting

- ✓ Previous work has been done in: [Solé,Salembier 2004,2007]
  - Lossless coding
  - 1-dimensional separable context
  - Applied directly in the Image domain

Lifting structures: Generalized lifting

- Previous work has been done in: [Solé,Salembier 2004,2007]
  - Lossless coding
  - 1-dimensional separable context
  - Applied directly in the Image domain

- Our main goal is to evaluate the potential of the GL method for lossy coding
  - Quantization
  - Comparison between 1-dimensional context and 2-dimensional non-separable context
  - Applied in the wavelet domain (as in bandelets)
  - *Assumption*: the pdf of the signal is known in the decoder (as well as in the coder)

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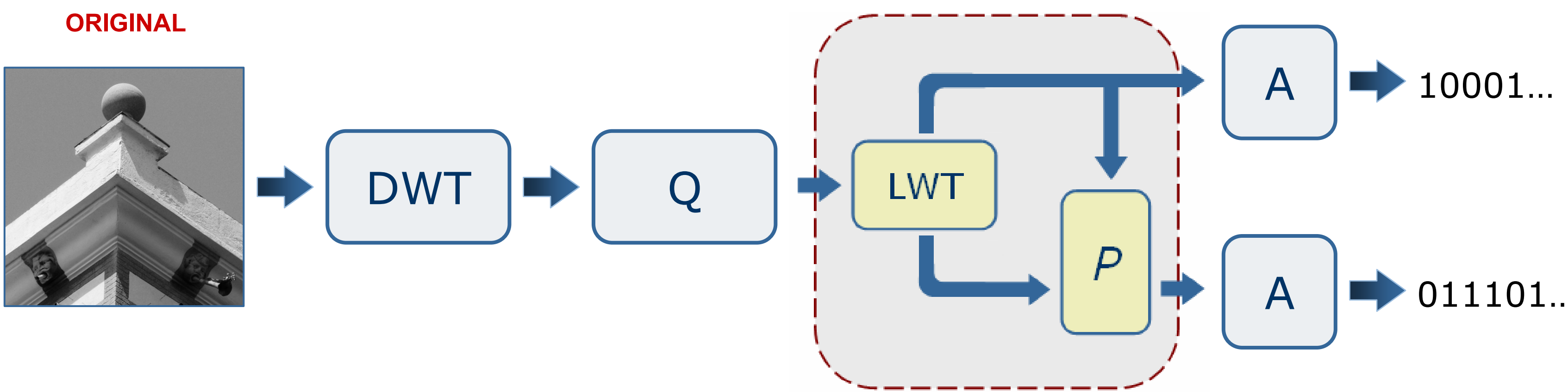
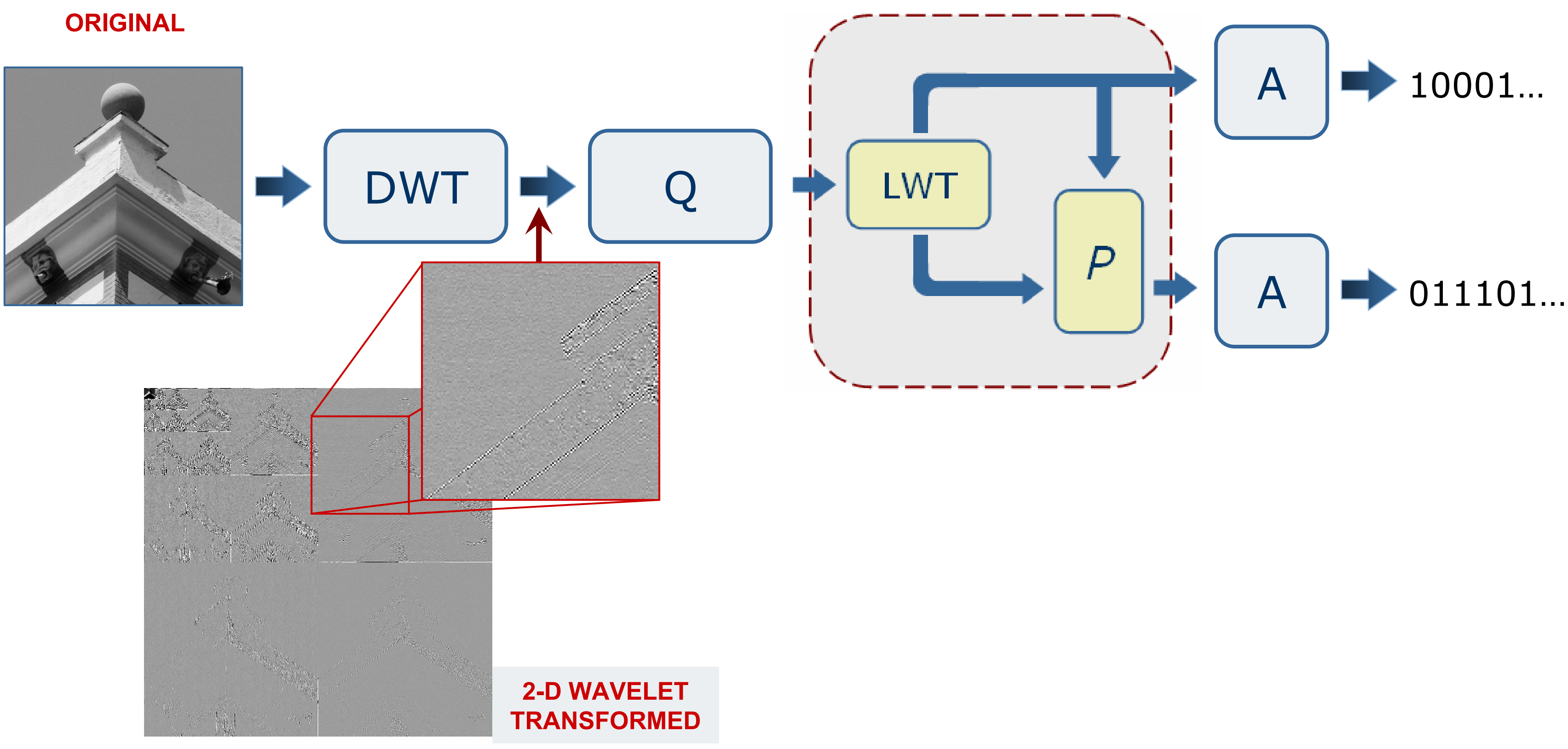
Coding scheme: The encoder



Coding scheme: The encoder



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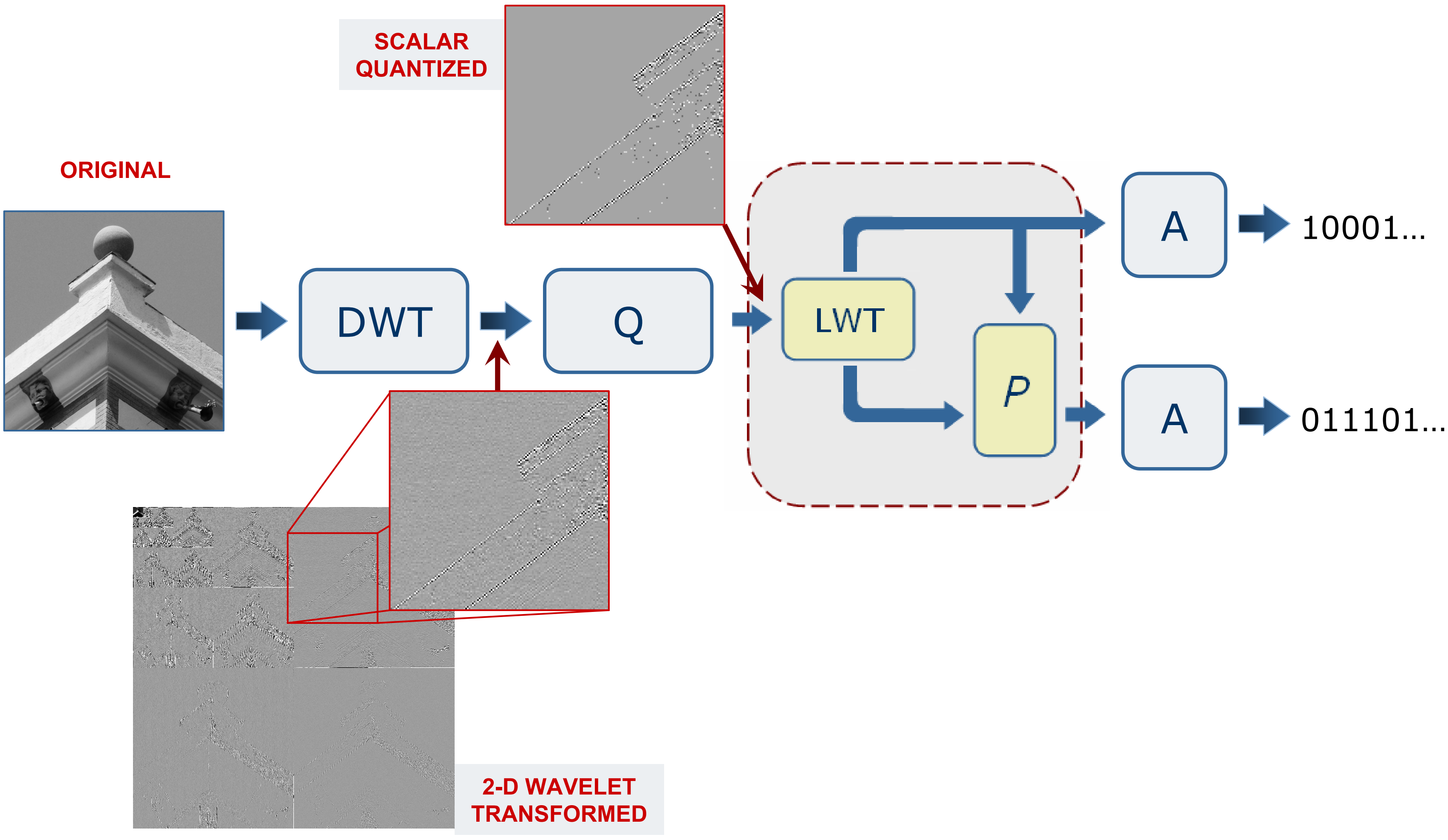

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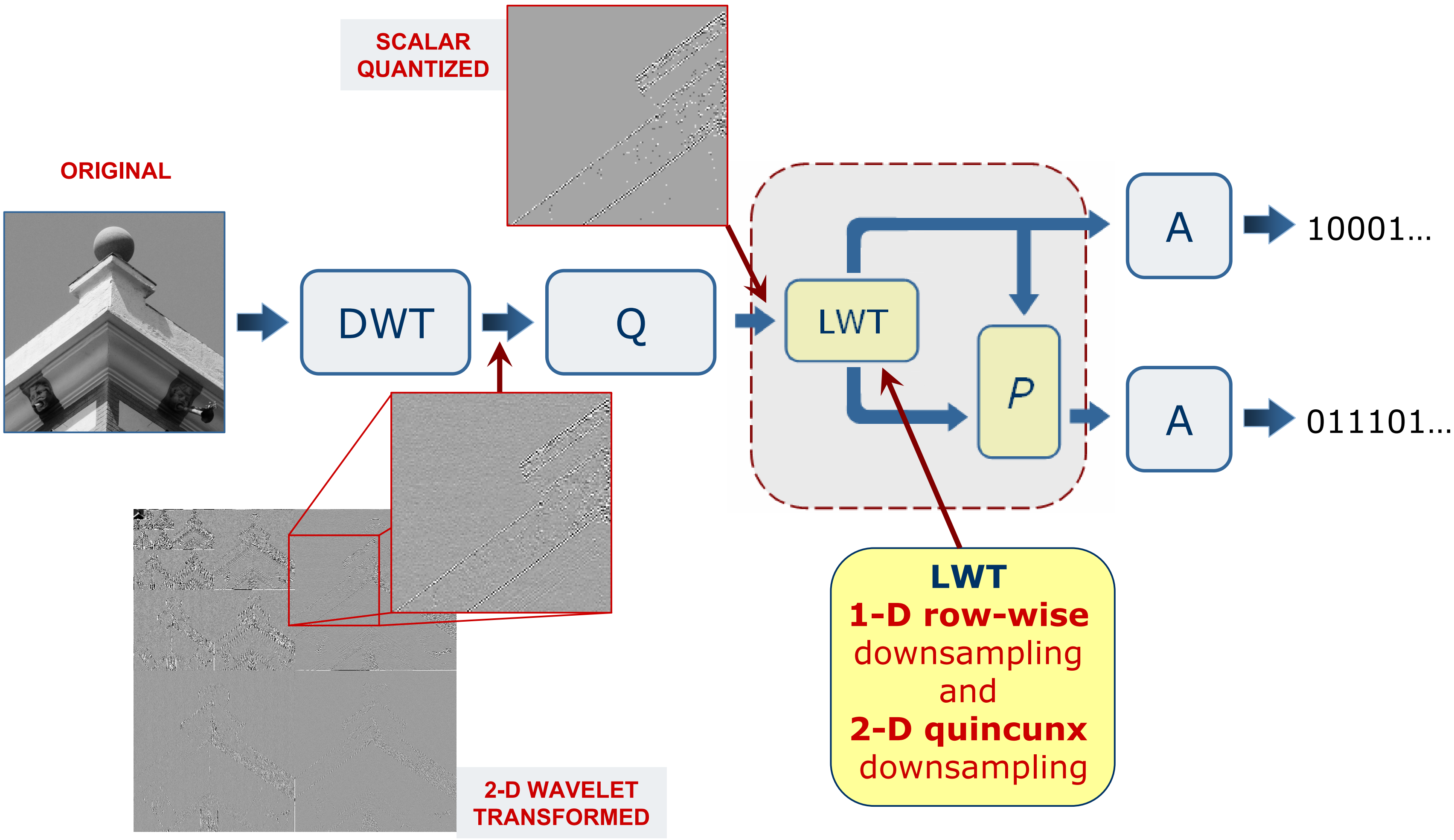

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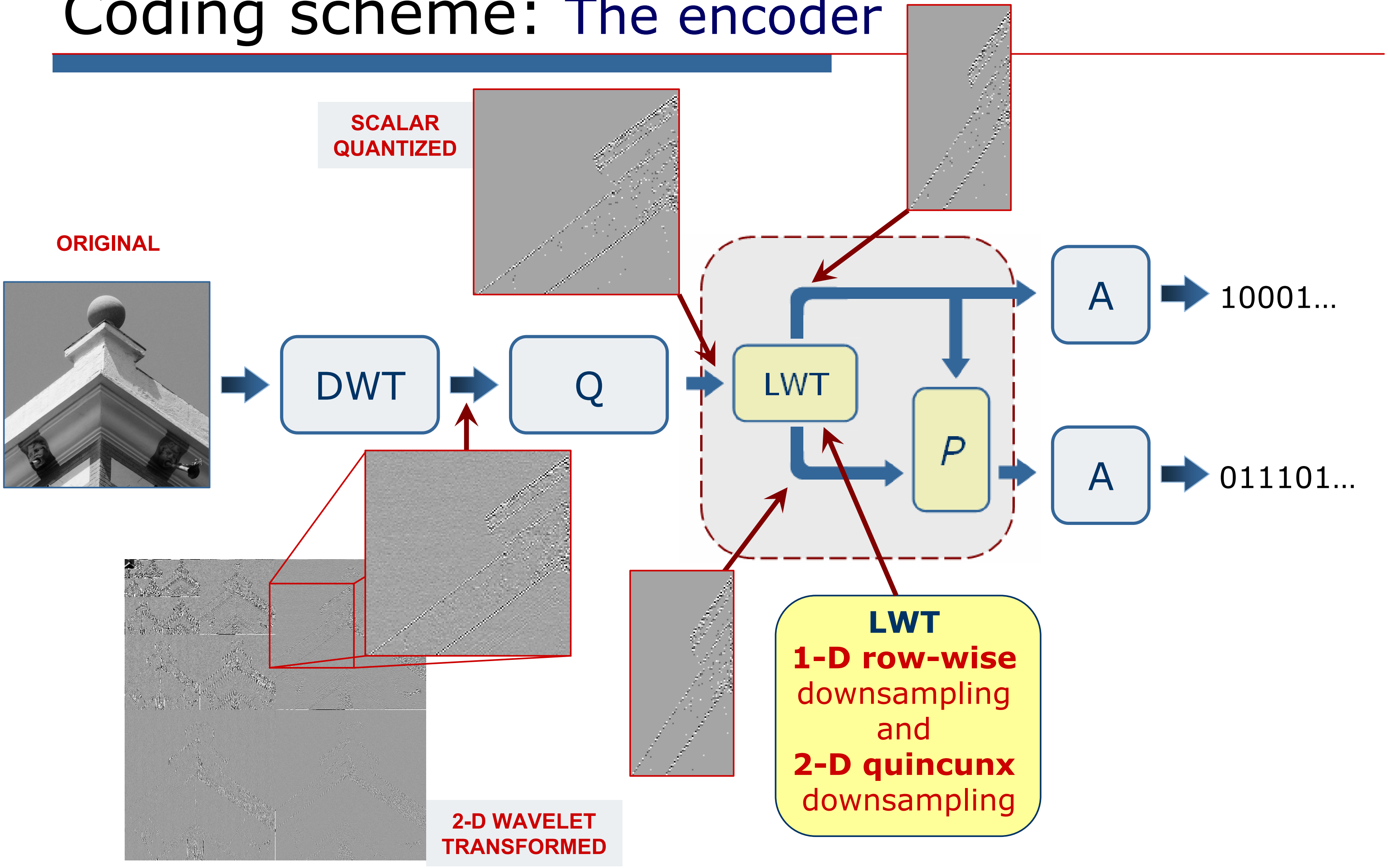

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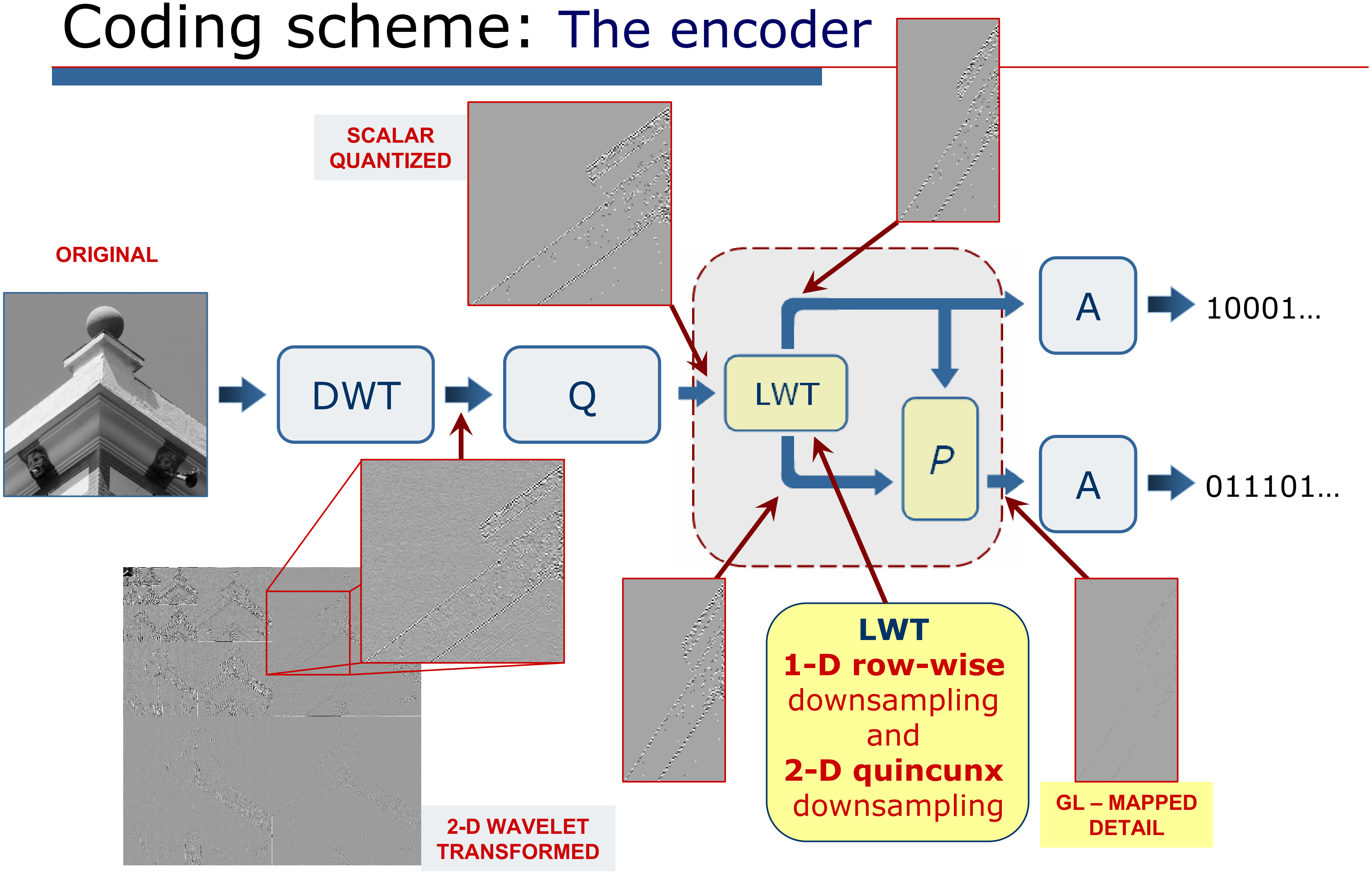
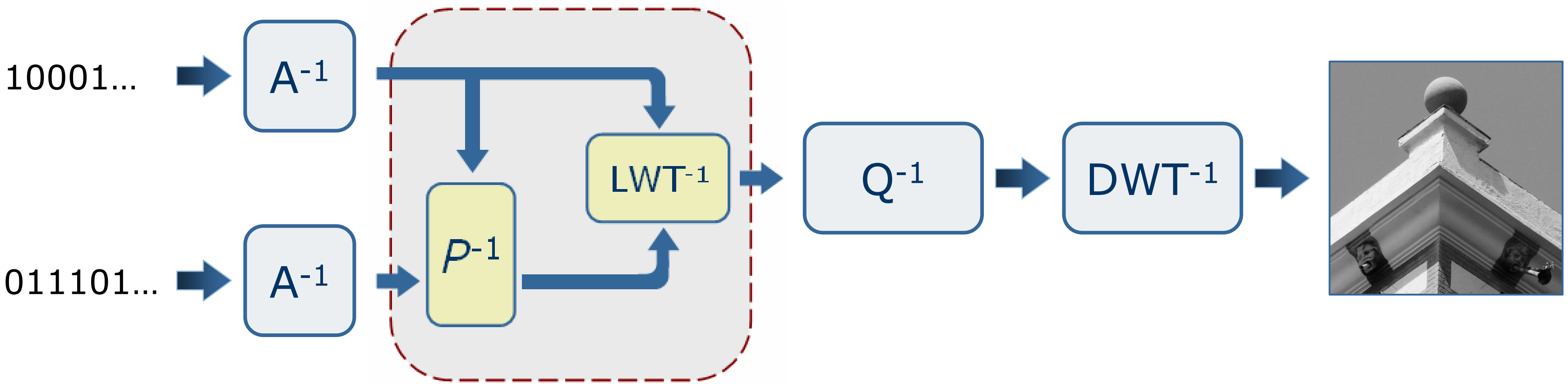
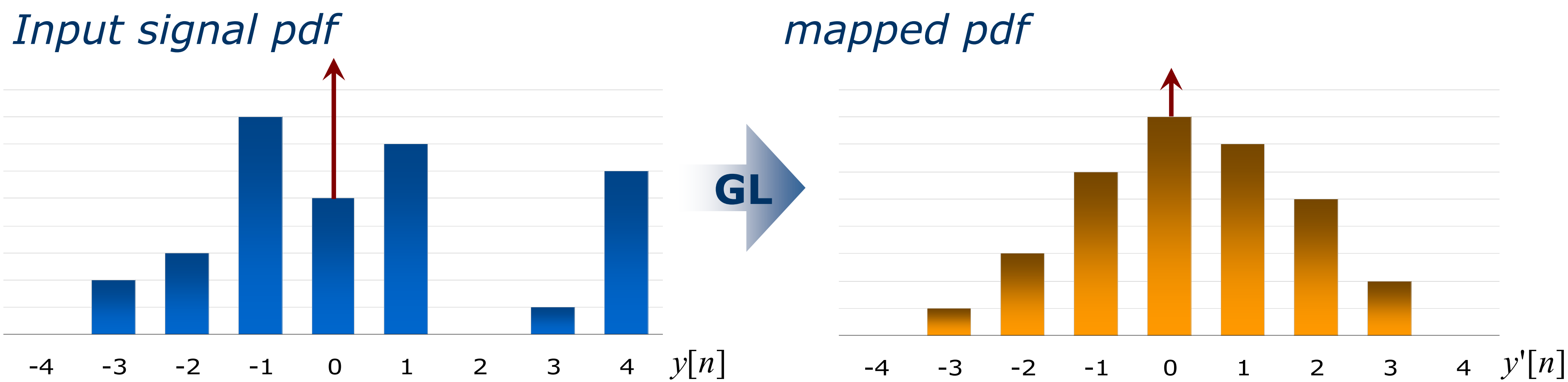


Coding scheme: The decoder

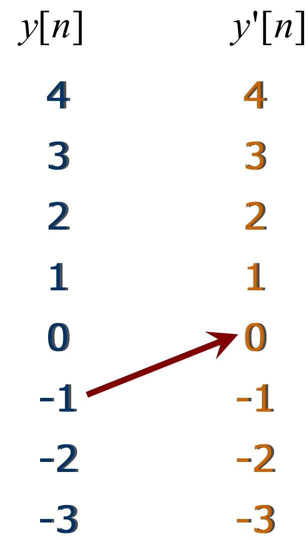


Coding scheme: Generalized predict design

Criterion: Energy minimization of the mapped details



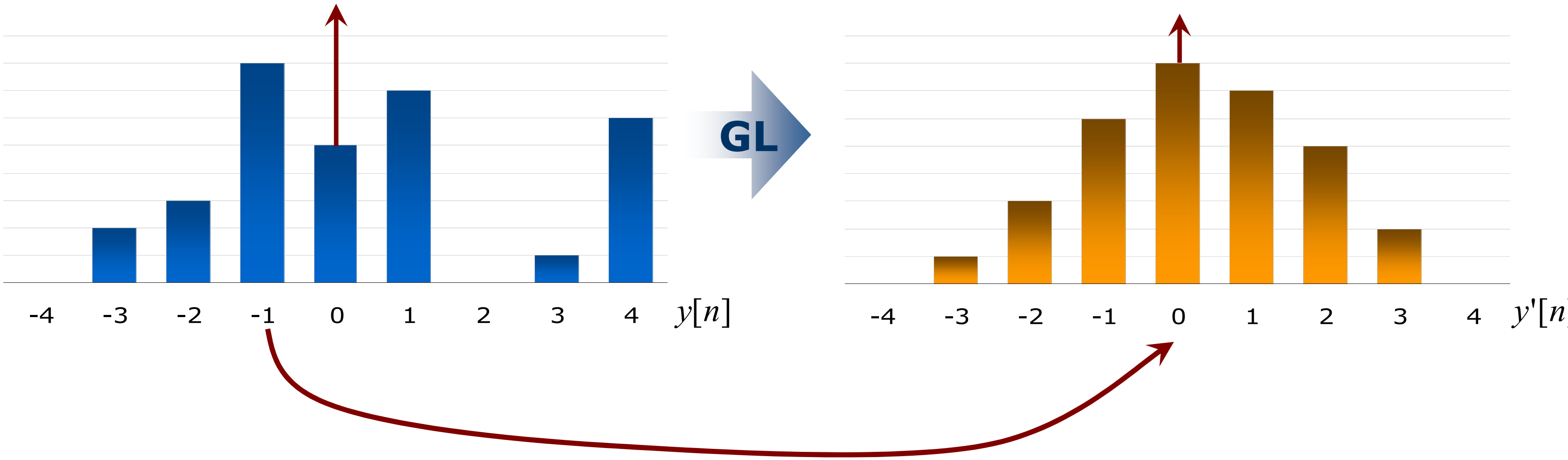
Coding scheme: Generalized predict des



Criterion: Energy minimization of the mapped details

Input signal pdf

mapped pdf



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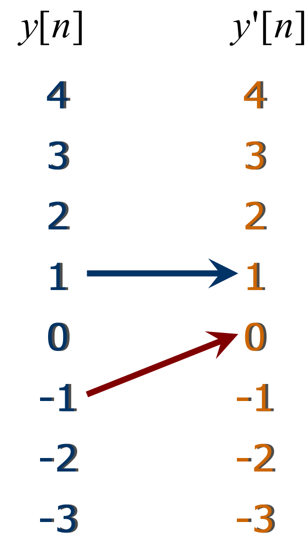

Criterion: **Energy minimization of the mapped details**

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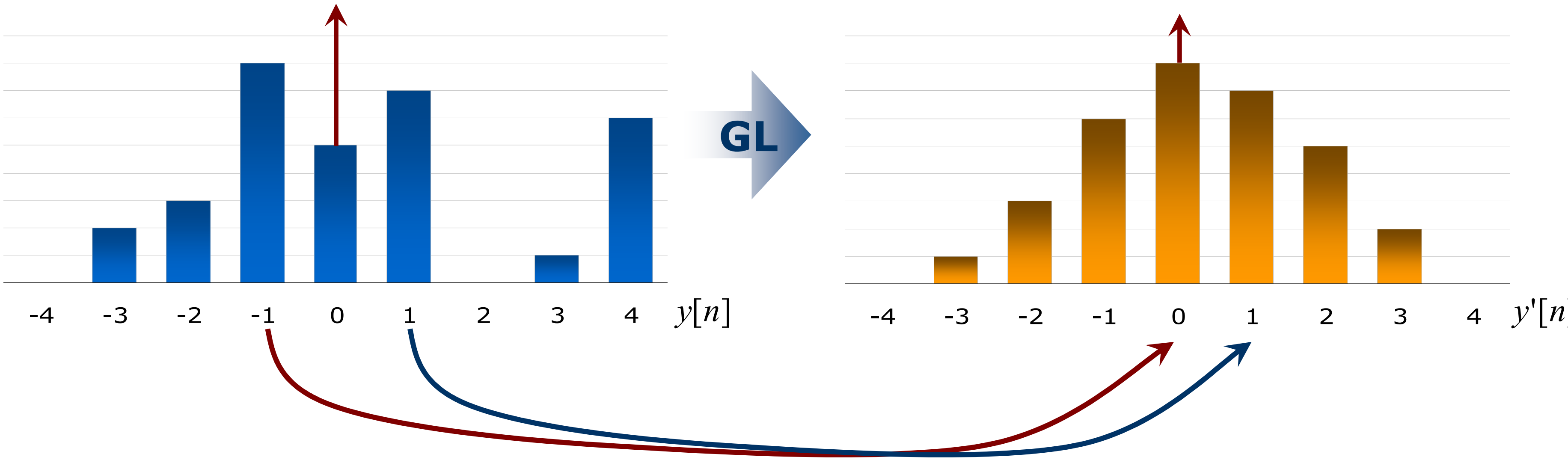
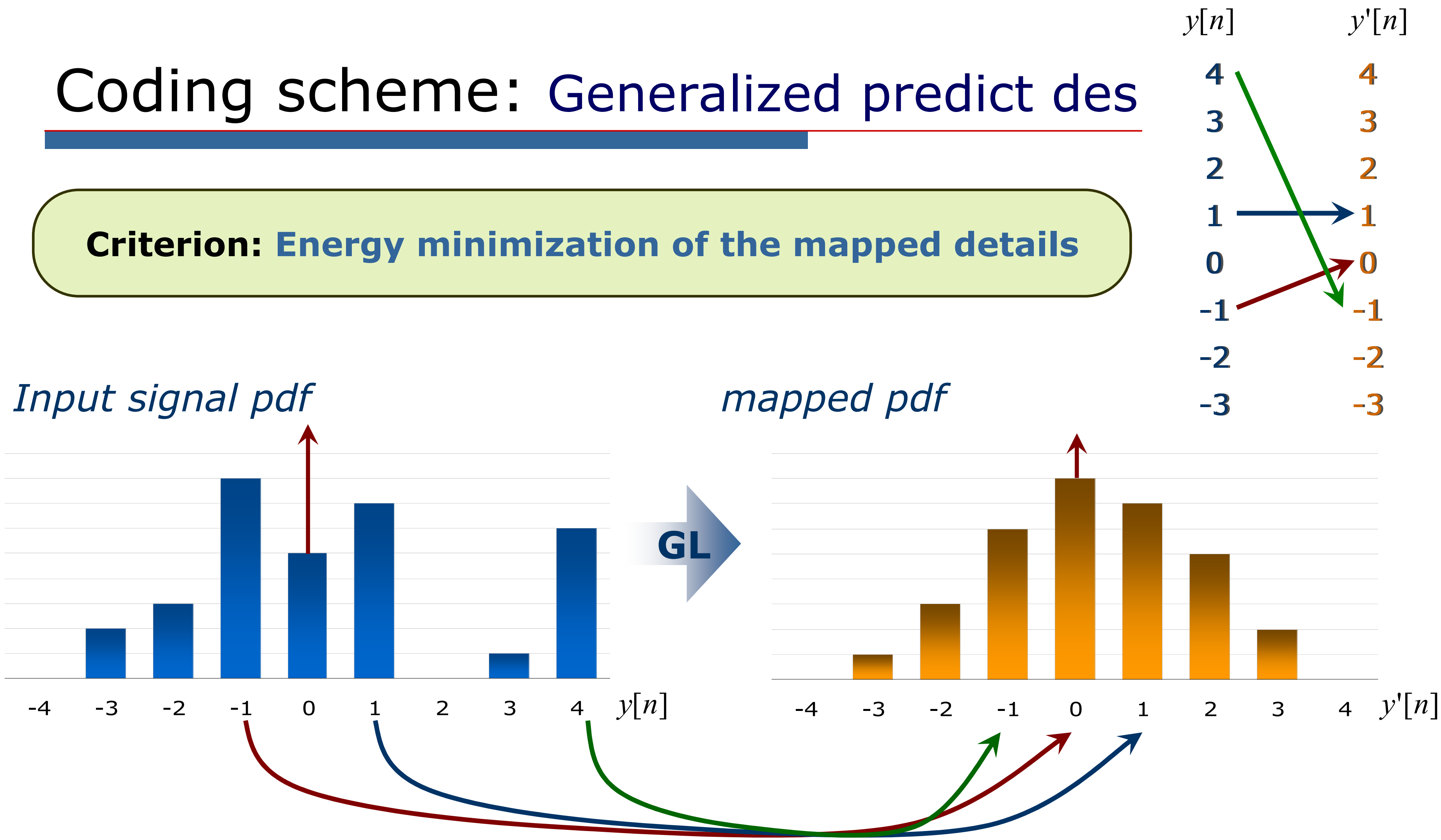


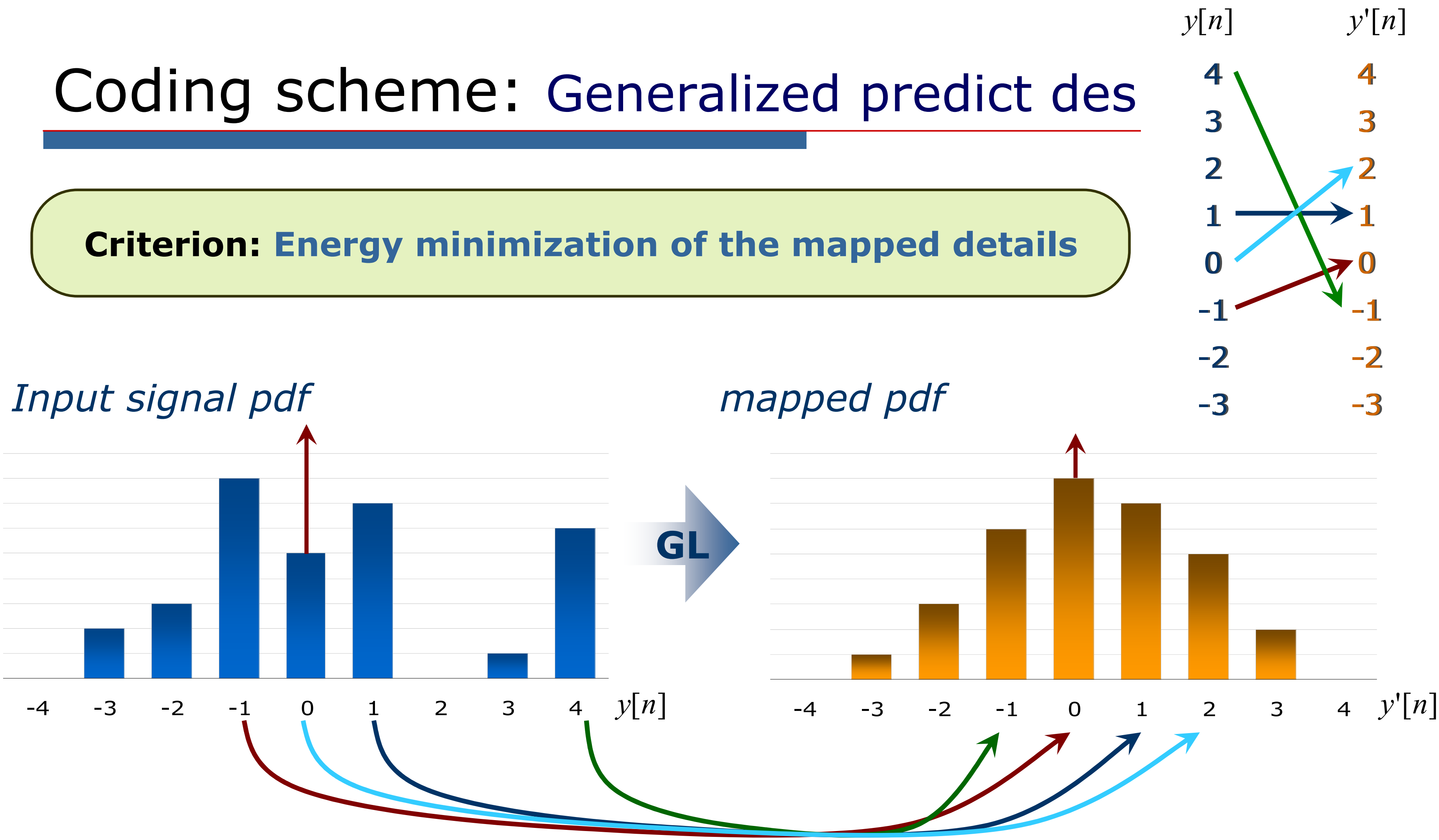
Coding scheme: Generalized predict design

Criterion: **Energy minimization of the mapped details**



Coding scheme: Generalized predict des

Criterion: Energy minimization of the mapped details



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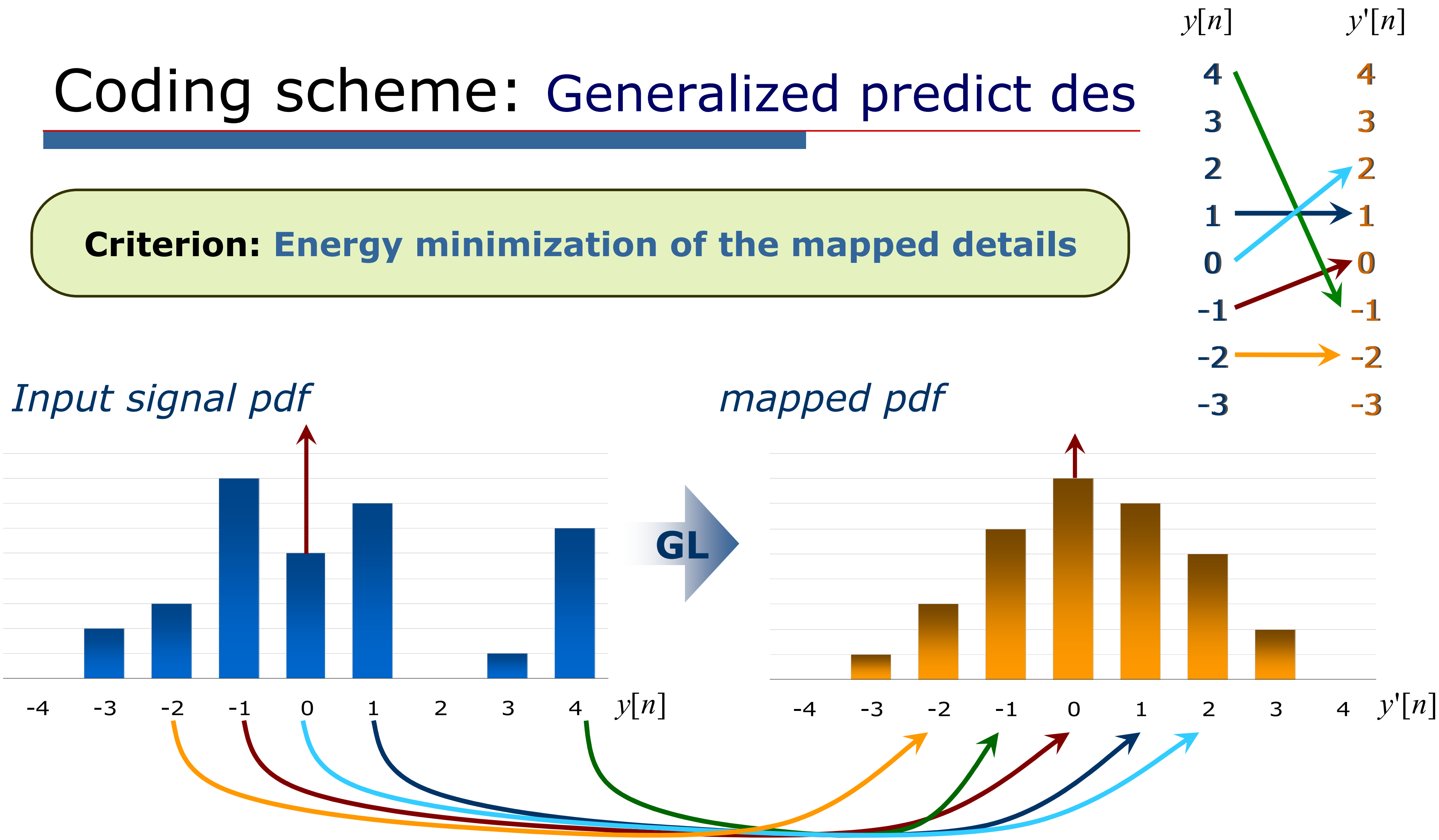

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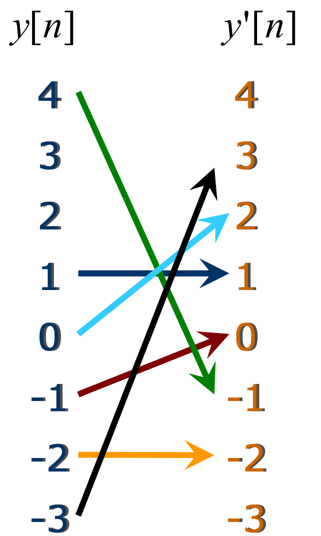

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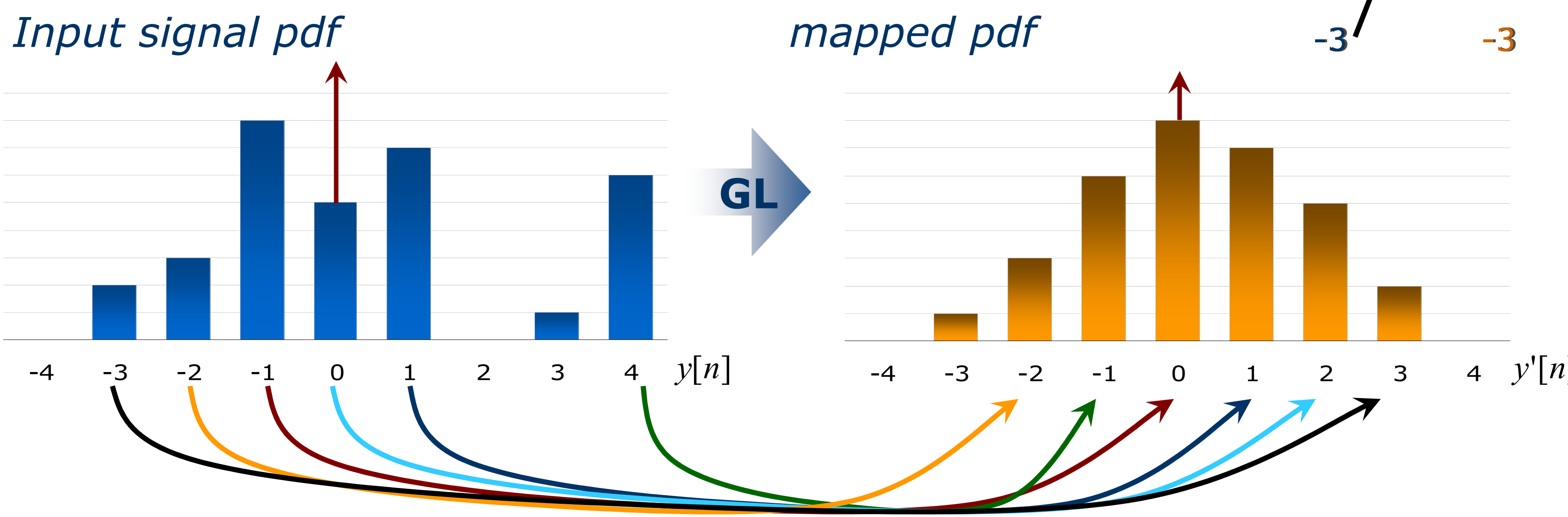

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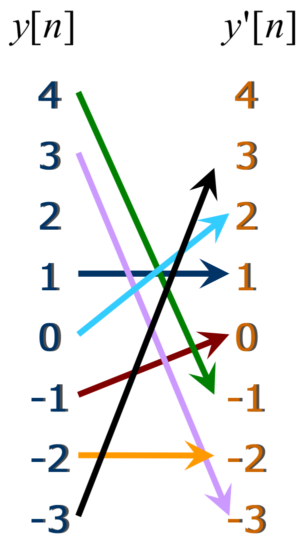

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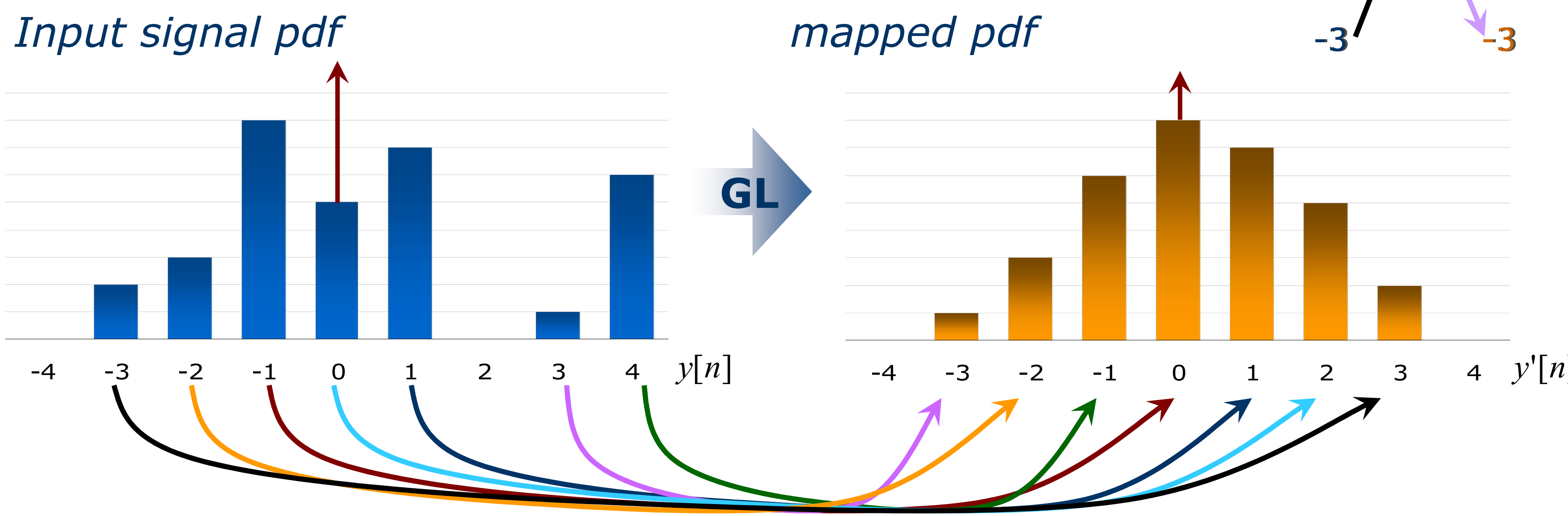

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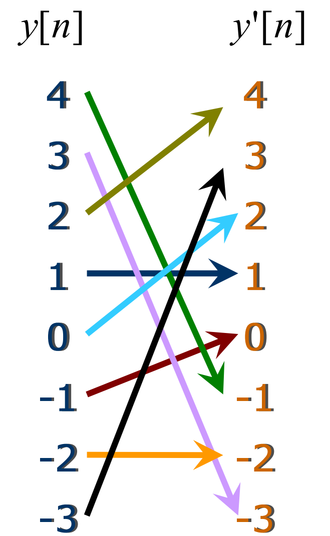

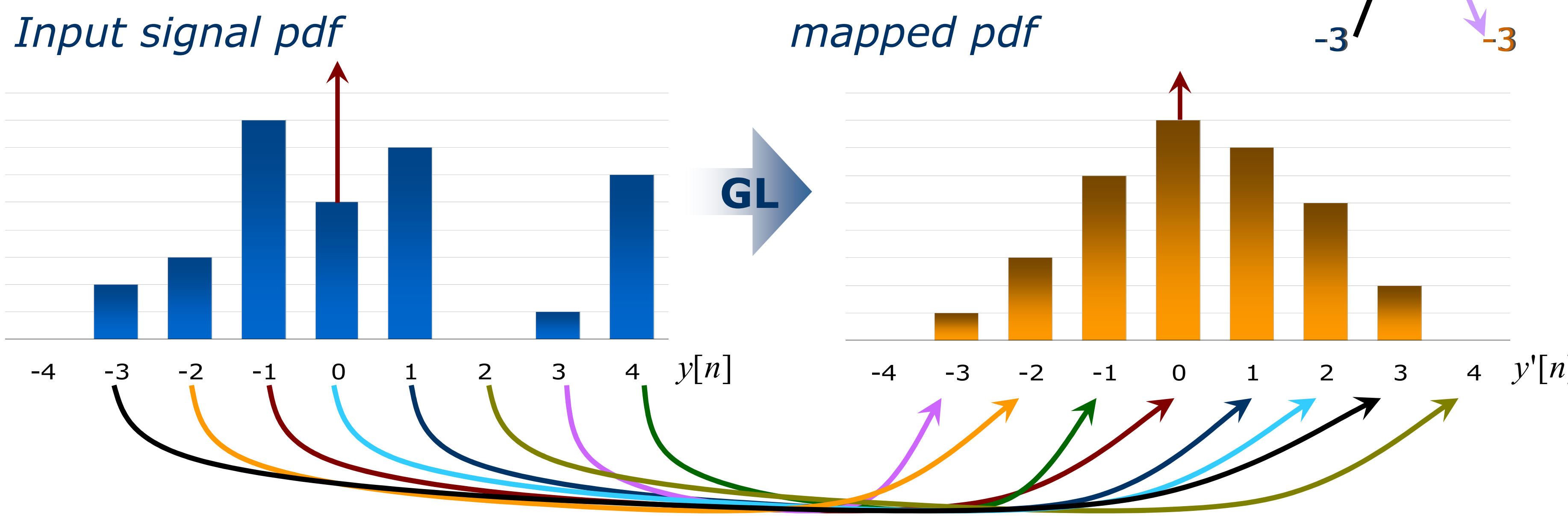

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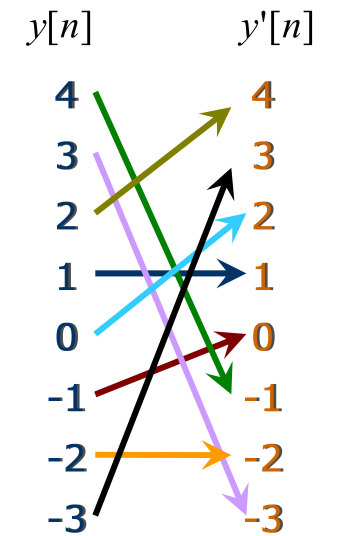

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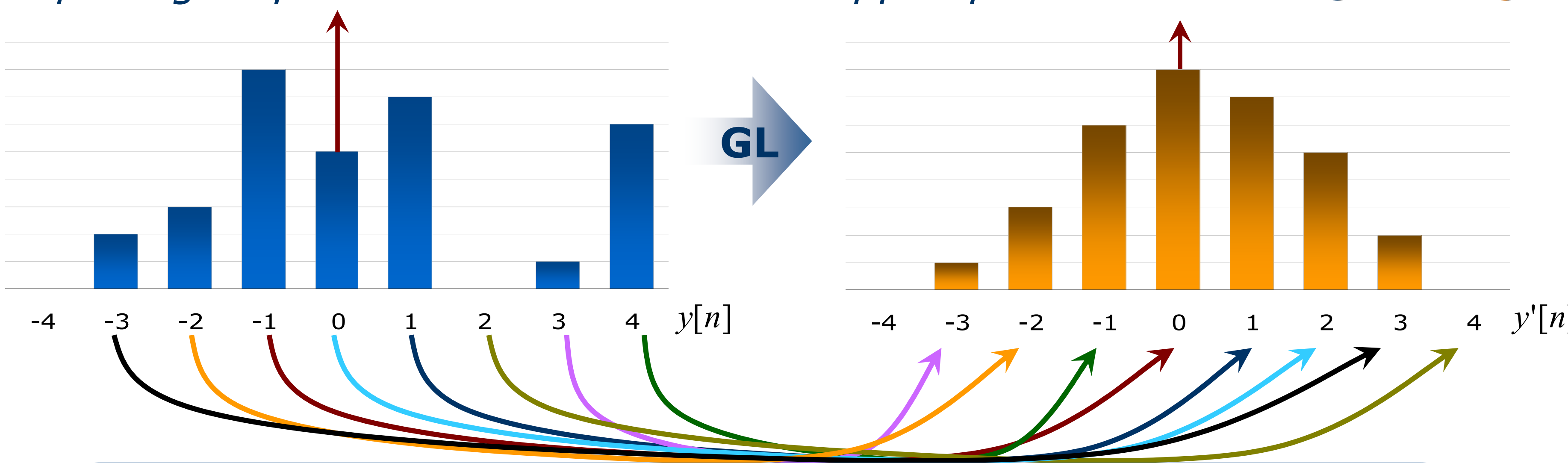

ASSUMPTION: Mapping is known by the decoder,
i.e. the decoder knows the pdf of the signal

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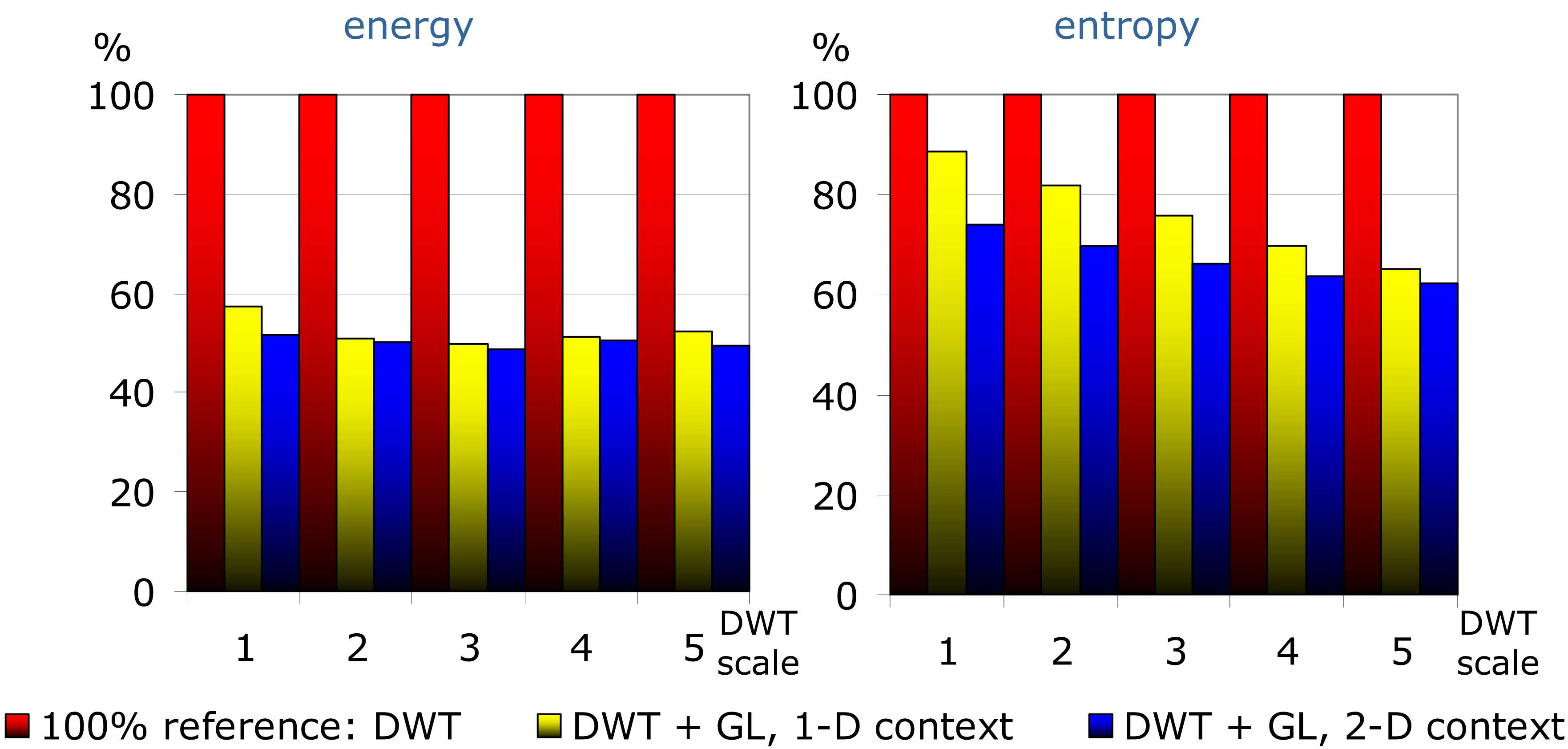
Experimental results: Energy and entropy



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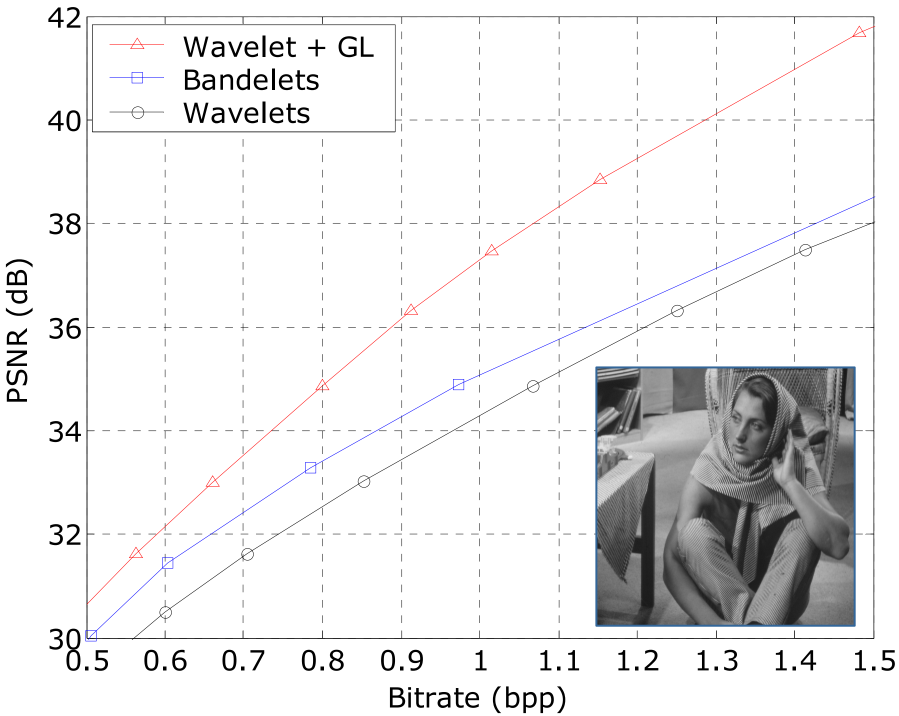
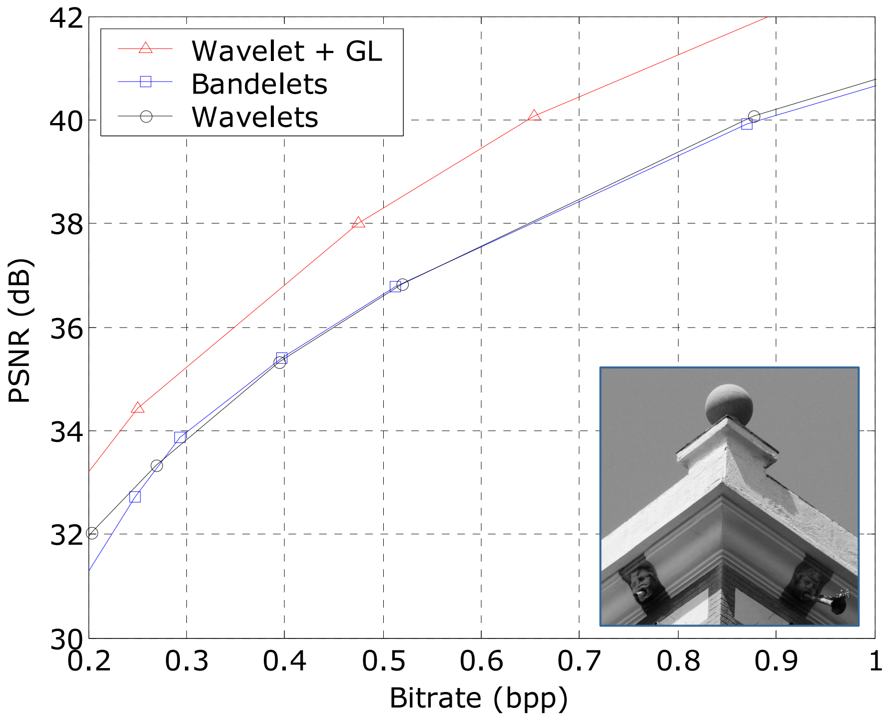
Experimental results: Rate-distortion

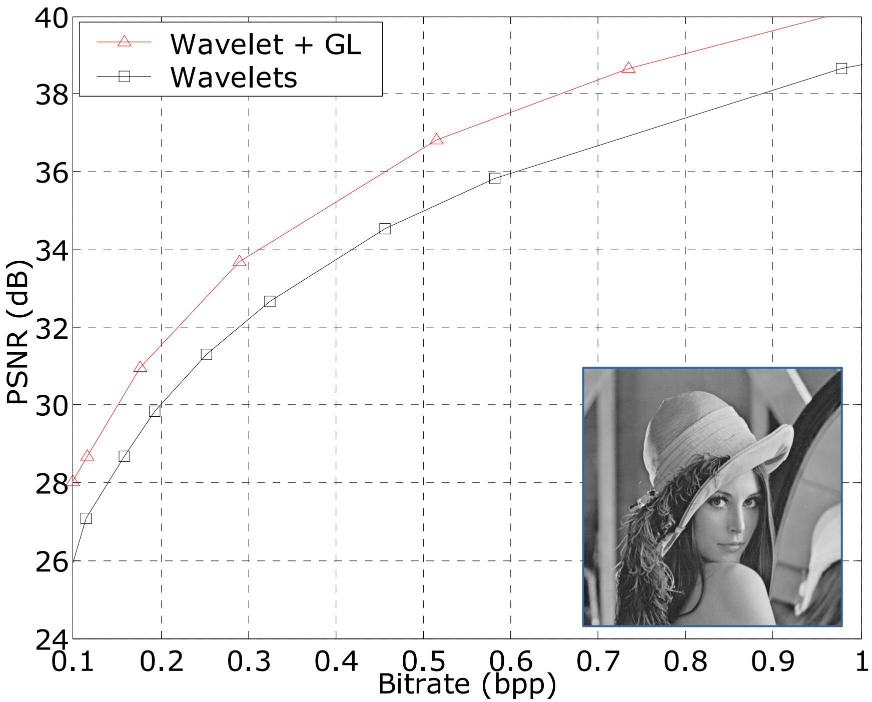
Generalized lifting
Potential gain is ~1-3 db, for 2-D context

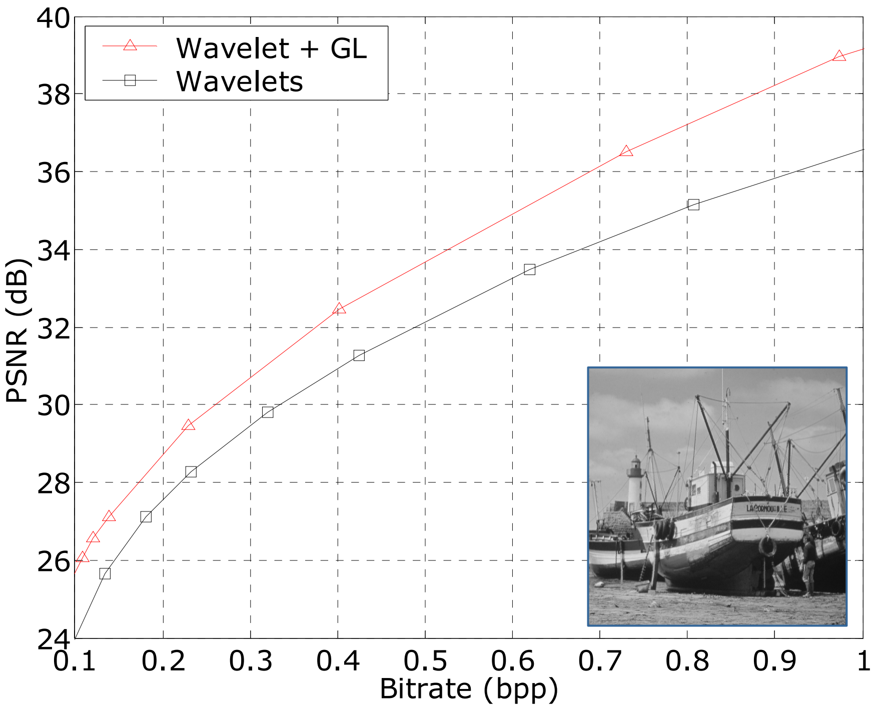




Experimental results: Rate-distortion

✓ Additional comparisons against wavelets





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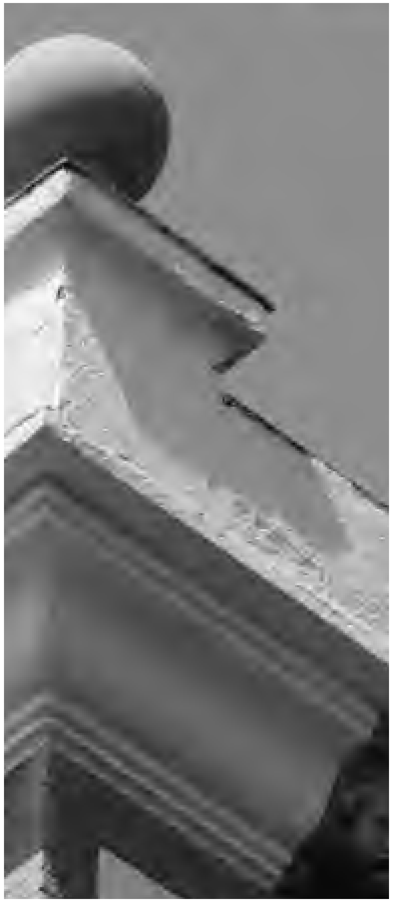
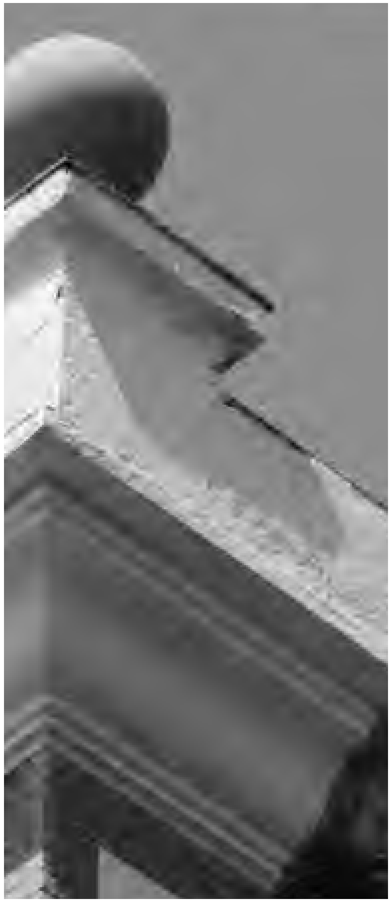
Experimental results: Perceptual quality @ 0.2bpp

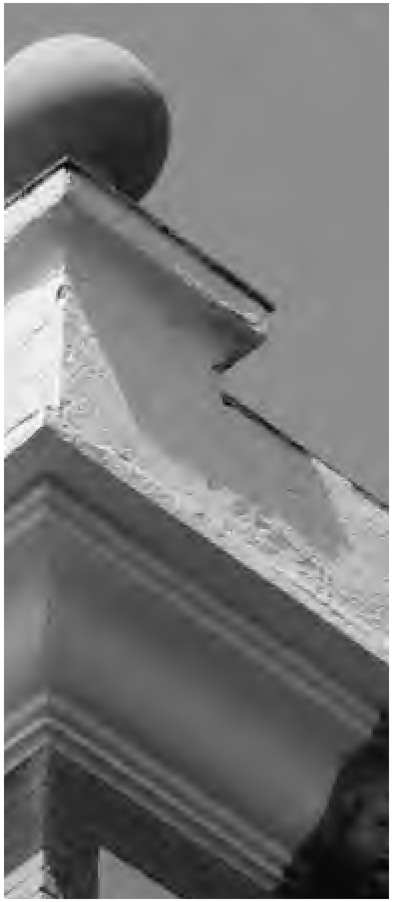
original



wavelets
32.03 dB

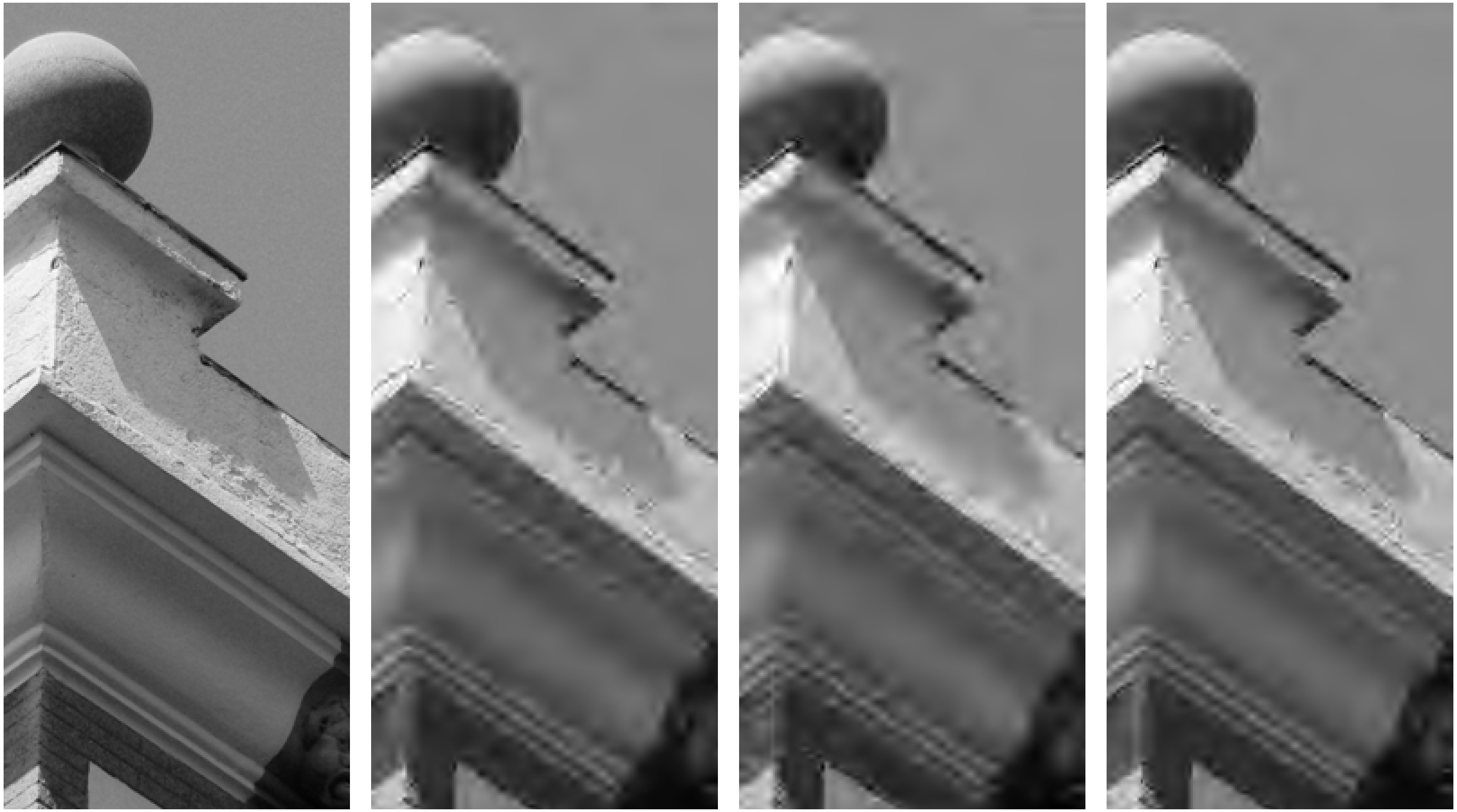


Bandelets
31.24 dB



GL
33.33 dB

Experimental results: Perceptual quality @ 0.11bpp



original | wavelets 29.47 dB | Bandelets 28.52 dB | GL 30.63 dB

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  (bandelets and adaptive directional lifting)
- ✓ The GL method differentiates from these methods in that is a non-linear framework
- ✓ Our predict design is based on the *pdf* of the image
- ✓ Generalized lifting approach gives promising results at reducing the energy while improving the R-D performance in 1-3 dB

Conclusions: Future work

- ✓ Remove the assumption that the decoder knows the *pdf* of the signal
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  - *This work:* one scale of GL applied for 1-D and 2-D contexts
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  - *Future:* include Q in the GL mapping, post-quantization
- ✓ Efficient entropy coding
  - *This work:* arithmetic encoding
  - *Future:* embedded-block coders like EBCOT, SPECK, etc.

Generalized Lifting for Sparse Image Representation and Coding

Questions

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Generalized Lifting for Sparse Image Representation and Coding

Thank you !!!

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