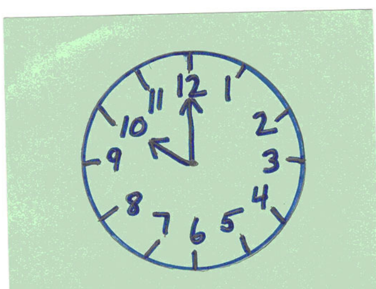
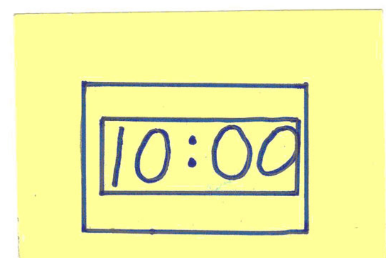
FROM ANALOGUE TO DIGITAL: CONCEPTS AND TECHNIQUES





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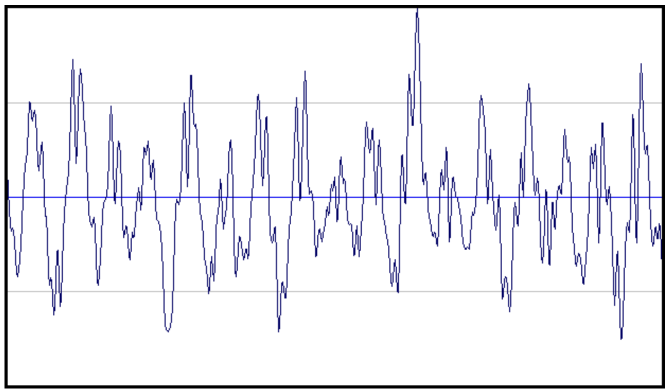
An Analogue World …



An analog/analogue signal is any variable signal, continuous in both time and amplitude.

- **Any information may be conveyed by an analogue signal; often such a signal is a measured response to changes in physical phenomena, such as sound or light, and is achieved using a transducer, e.g. camera or microphone.**
- **A disadvantage of analogue representation is that any system has noise—that is, random variations—in it; as the signal is transmitted over long distances, these random variations may become dominant.**

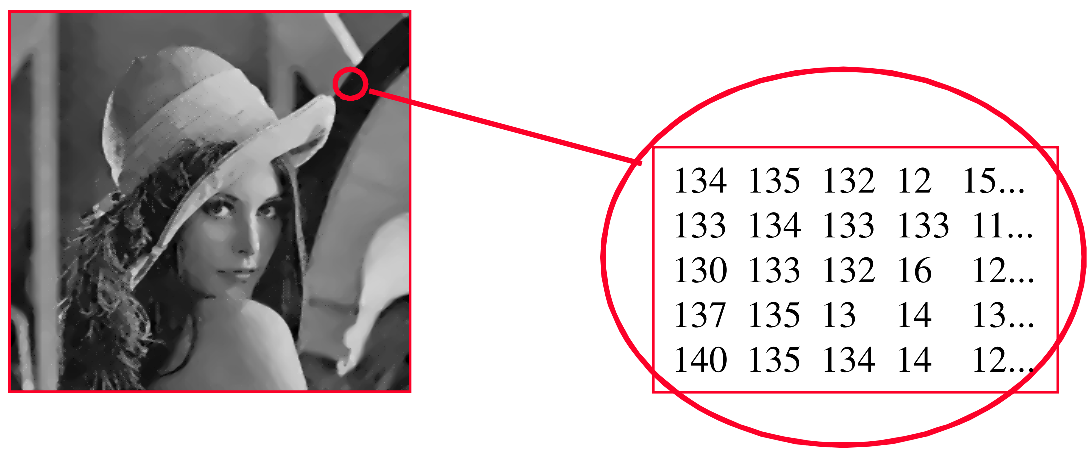
Digitization

Process of expressing analogue data in digital form.

Analogue data implies ‘continuity’ while digital data is concerned with discrete states, e.g. symbols, digits.

Vantages of digitization:

- **Easier to process**
- **Easier to compress**
- **Easier to multiplex**
- **Easier to protect**
- **Lower powers**
- **...**



Sampling or Time Discretization

Sampling is the process of obtaining a periodic sequence of samples to represent an analogue signal.

Sampling is governed by the Sampling Theorem which states that:

An analog signal may be fully reconstructed from a periodic sequence of samples if the sampling frequency is, at least, twice the maximum frequency present in the signal.

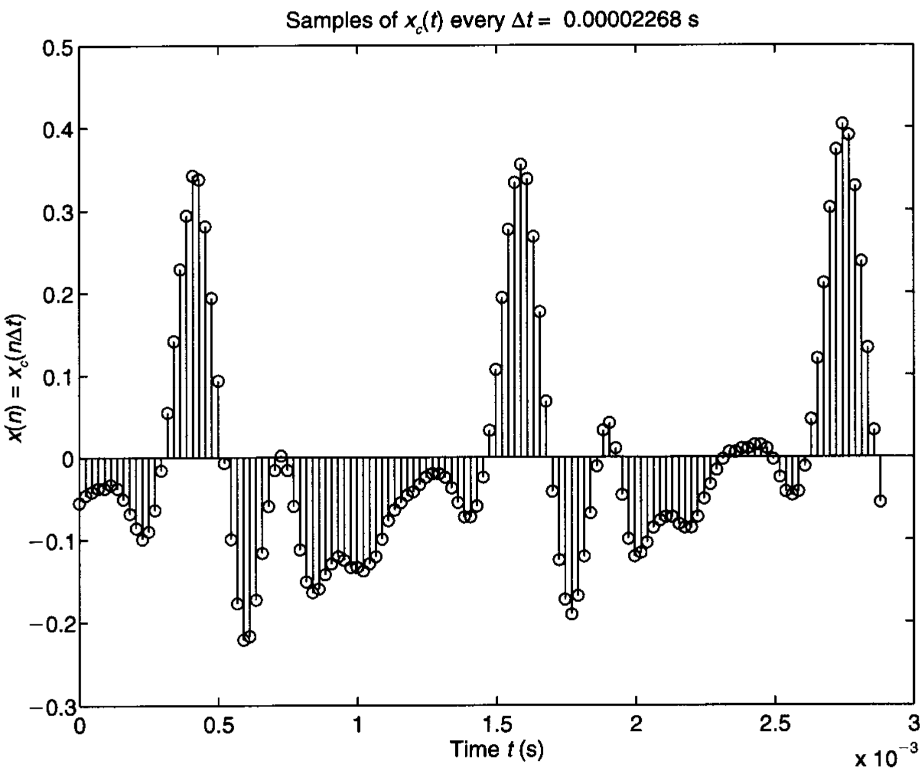
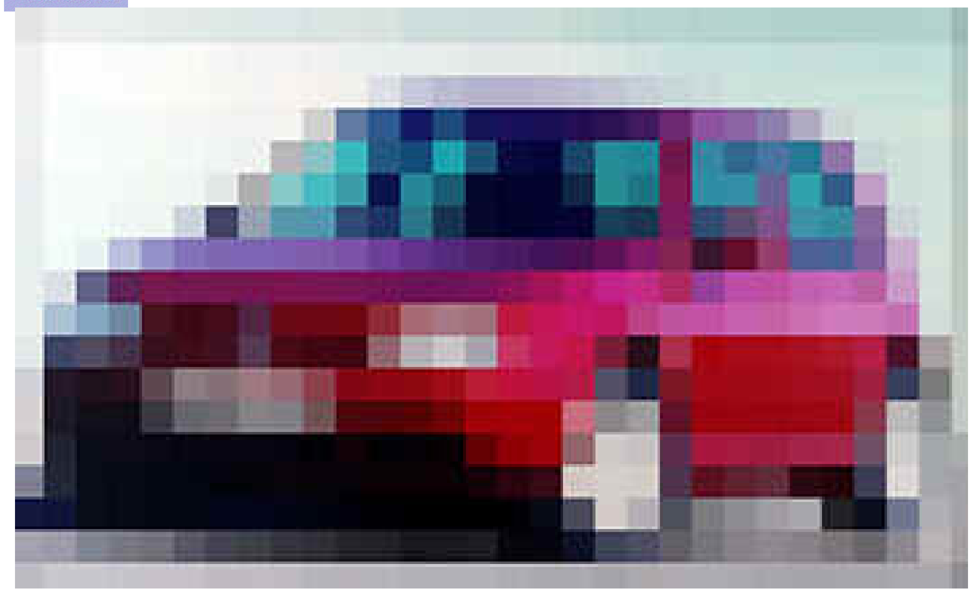
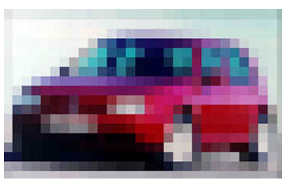
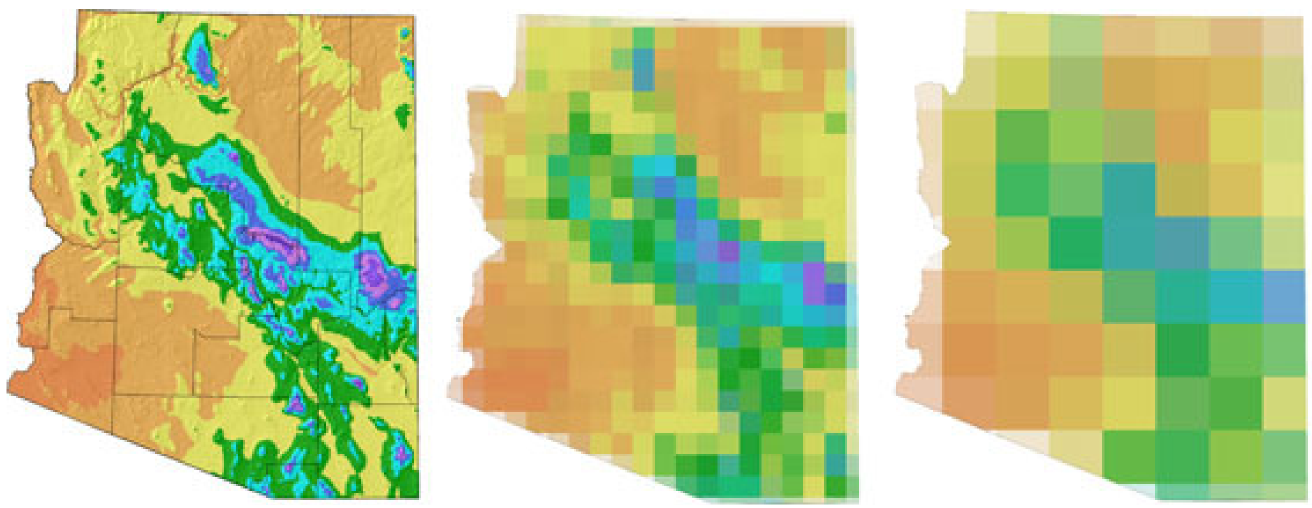


Image Sampling



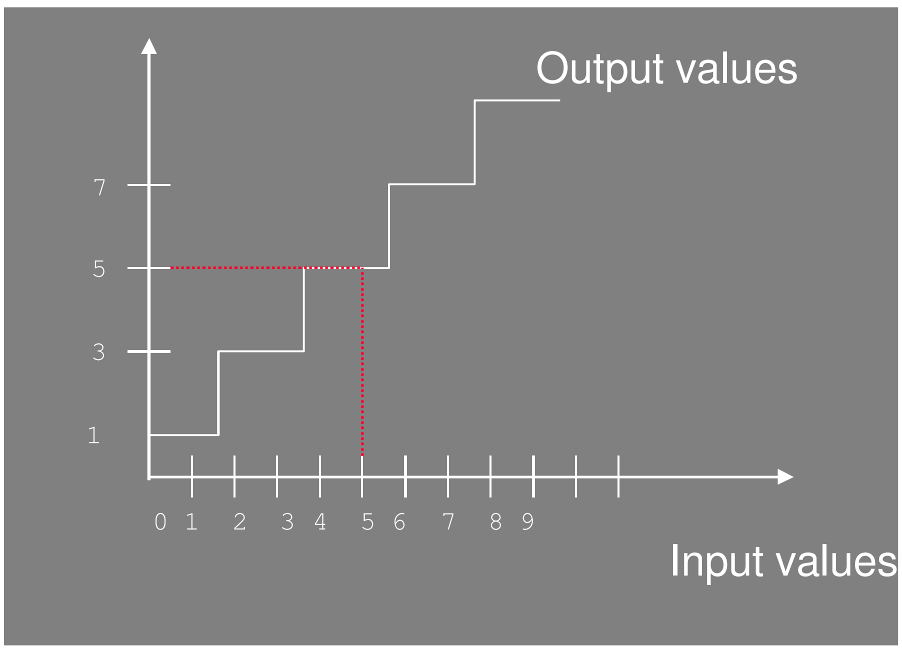
The number of samples (resolution) of an image is very important to determine the 'final quality'.

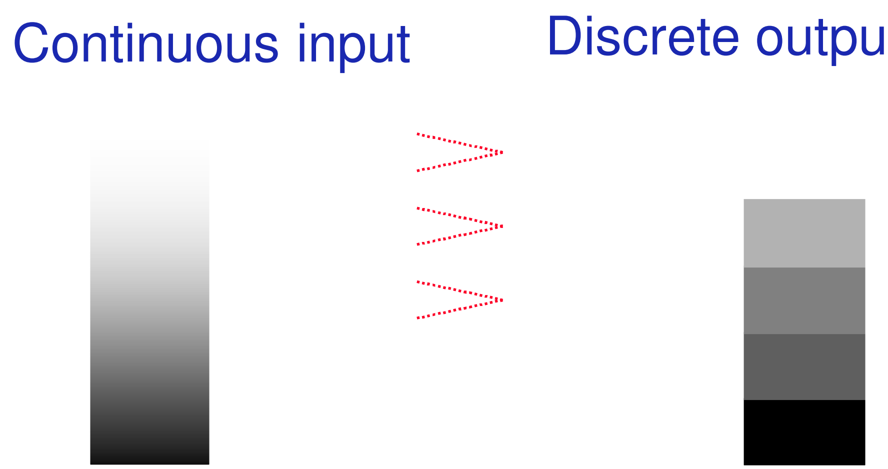




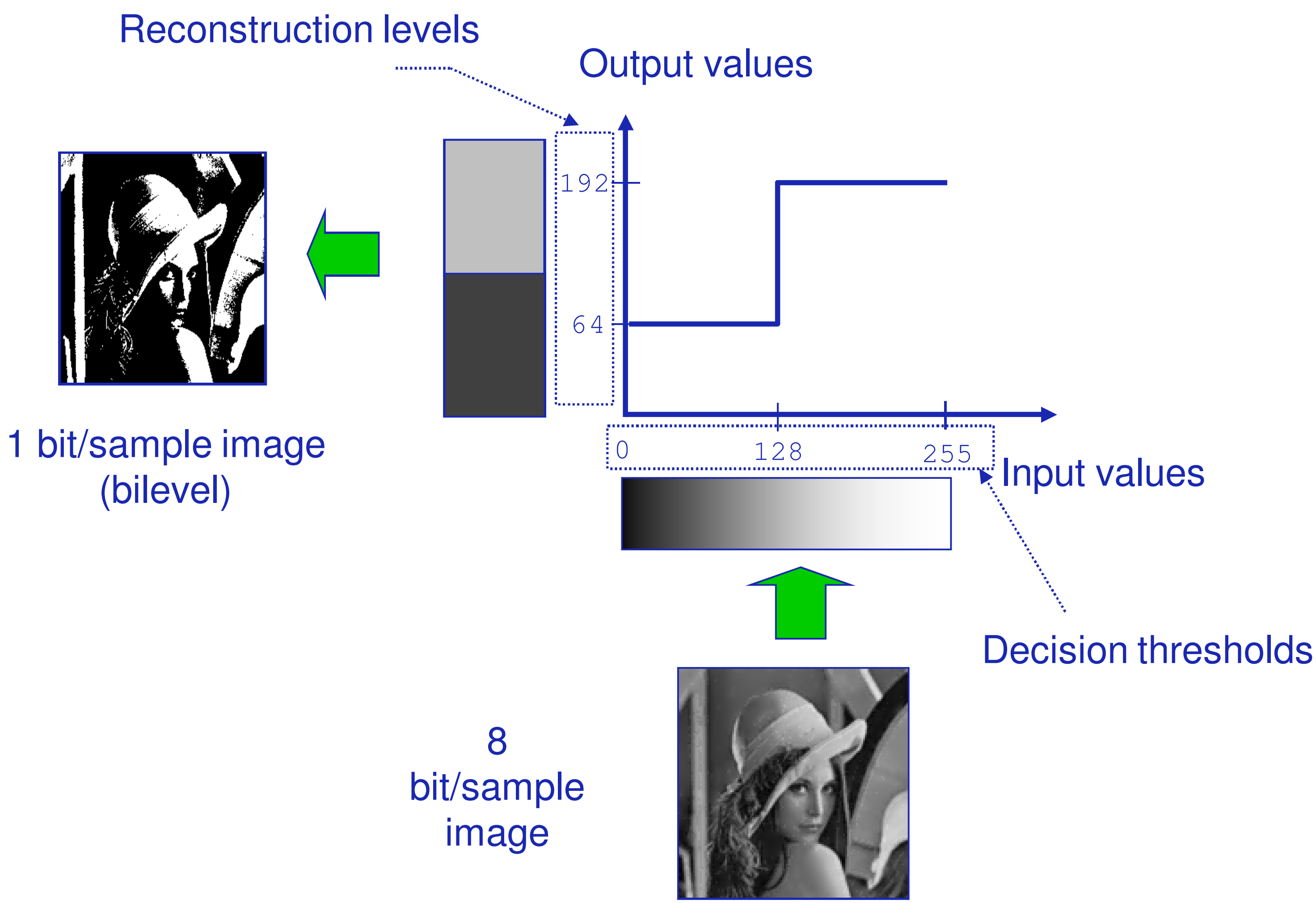
Quantization or Amplitude Discretization

Quantization is the process in which the continuous range of values of a sampled input analogue signal is divided into non-overlapping subranges, and to each subrange a discrete value of the output is uniquely assigned.

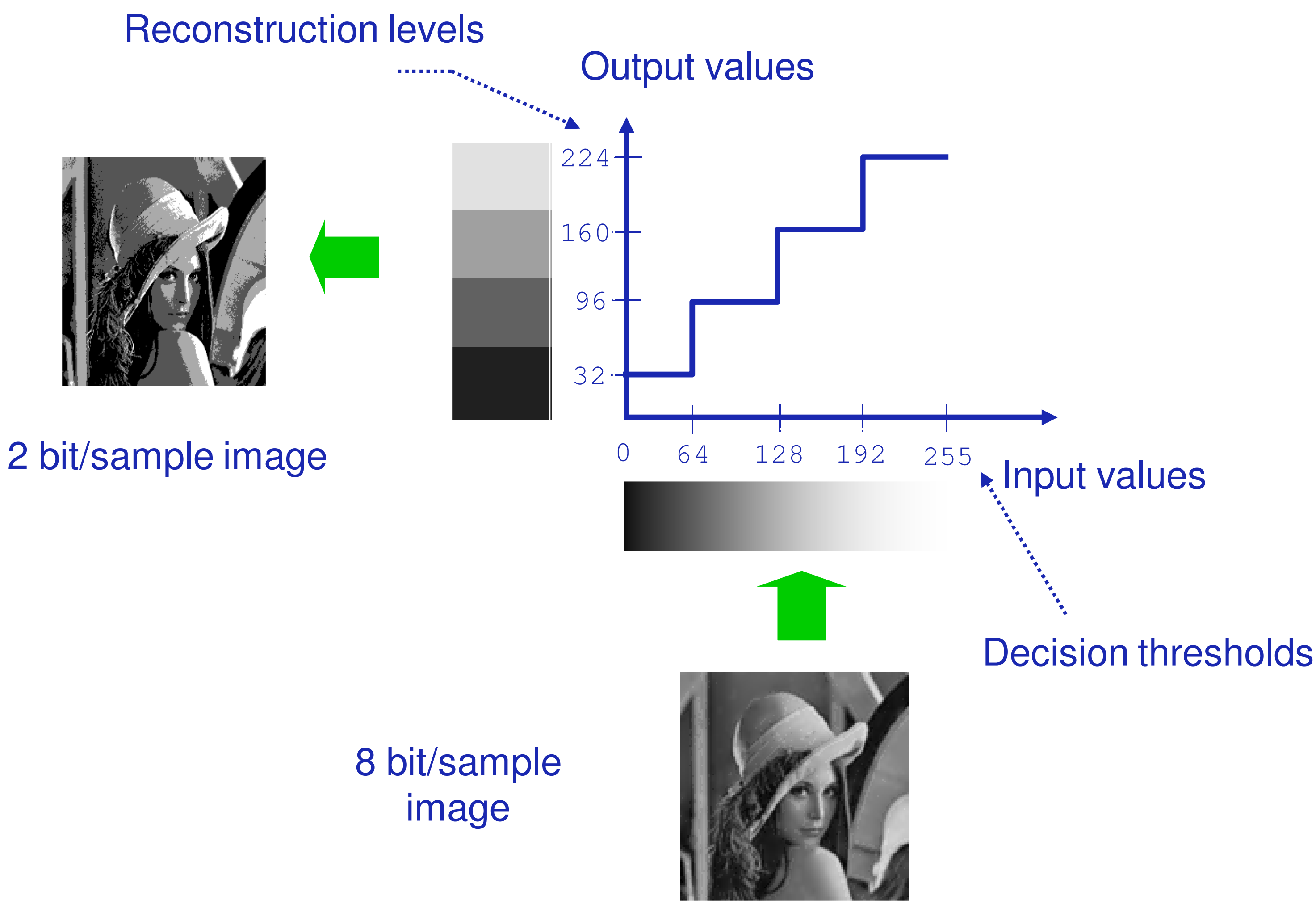




2 Levels Quantization



4 Levels Quantization



Uniform Quantization

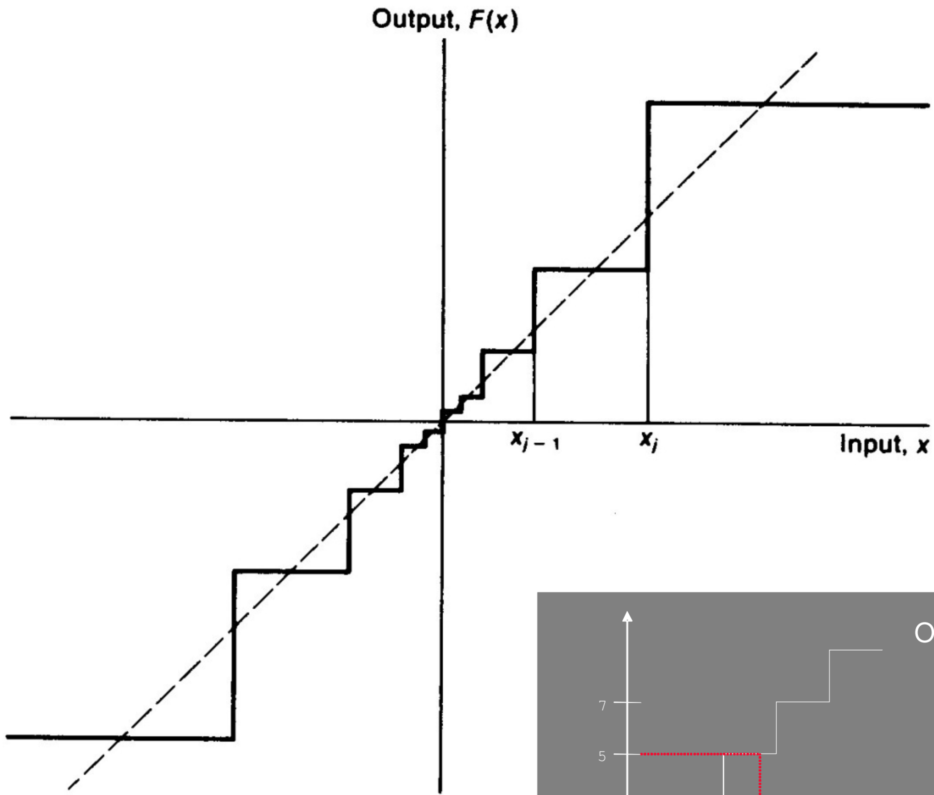
4 bit/sample
0000, 0001,
0010, 0011, …

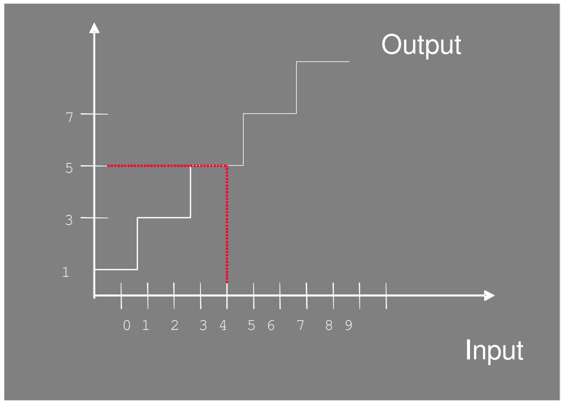
2 bit/sample
00, 01, 10 , 11

3 bit/sample
000, 001, 010,
011, 100, 101,
110, 111

1 bit/sample
0, 1

Non-Uniform Quantization

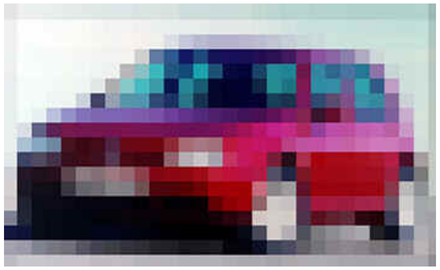




For many signals, e.g., speech, uniform or linear quantization is not a good solution in terms of minimizing the mean square error (and thus the Signal to Quantization noise Ratio, SQR) due to the non-uniform statistics of the signal.

Also to get a certain SQR, lower quantization steps have to be used for lower signal amplitudes and vice-versa.

Pulse Code Modulation (PCM)

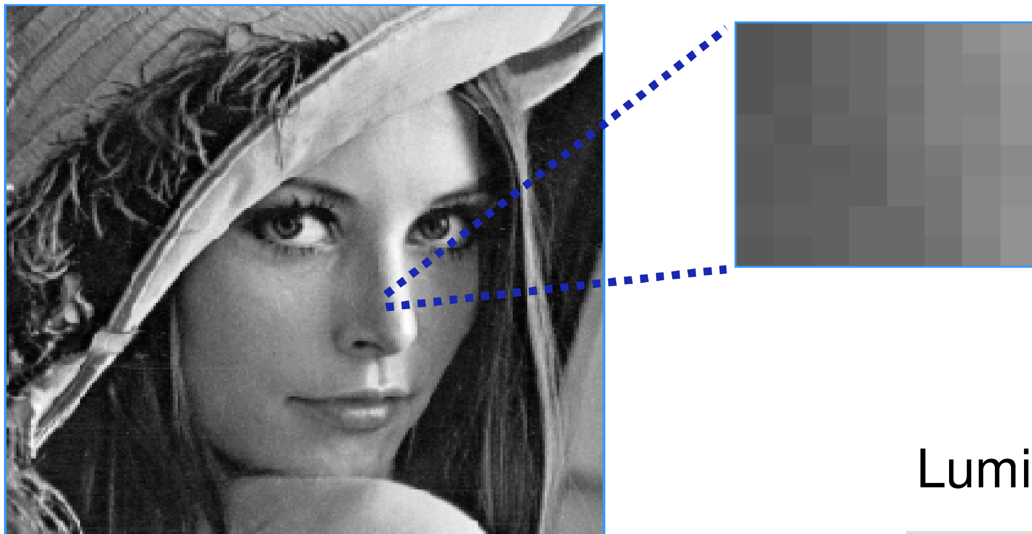


PCM is the simplest form of digital source representation/coding where each sample is <u>independently</u> represented with the same number of bits.

- **Example 1: Image with 200×100 samples at 8 bit/sample takes 200 × 100 × 8 = 160000 bits with PCM coding**
- **Example 2: 11 kHz bandwidth audio at 8 bit/sample takes 11000 × 2 × 8 = 176 kbit/s kbit/s with PCM coding**

Being the simplest form of coding, as well as the least efficient, PCM is typically taken as the reference/benchmark coding method to evaluate the performance of more powerful (source) coding algorithms.

Image, Samples and Bits …



Luminance =

87	89	101	106	118	130	142	155
85	91	101	105	116	129	135	149
86	92	96	105	112	128	131	144
92	88	102	101	116	129	135	147
88	94	94	98	113	122	130	139
88	95	98	97	113	119	133	141
92	99	98	106	107	118	135	145
89	95	98	107	104	112	130	144

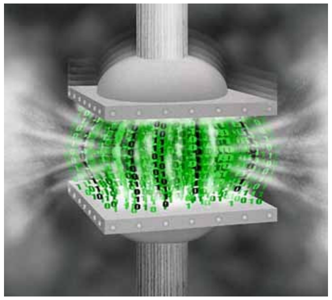
Binary representation
8 bit/sample -> 256 (2^8) levels

87 = **0101 0111**
130 = **1000 0010**

Why Compressing ?

- **Speech – e.g. 8000 samples/s with 8 bit/sample – 64000 bit/s = 64 kbit/s**
- **Music – e.g. 44000 samples/s with 16 bit/sample – 704000 bit/s=704 kbit/s**
- **Standard Video – e.g. (576×720+2×576×360)×25 (20736000) samples/s with 8 bit/sample – 166000000 bit/s = 166 Mbit/s**
- **Full HD 1080p - (1080×1920+2×1080×960)×25 (103680000) samples/s with 8 bit/sample – 829440000 bit/s = 830 Mbit/s**

How Much is Enough ?



- **Recommendation ITU-R 601: 25 images/s with 720×576 luminance samples and 360×576 samples for each chrominance with 8 bit/sample**

$$[(720\times576) + 2 \times (360 \times 576)] \times 8 \times 25 = 166 \text{ Mbit/s}$$

- **Acceptable rate, p.e. using H.264/MPEG-4 AVC: 2 Mbit/s**

=> Compression Factor: 166/2 ≈ 80

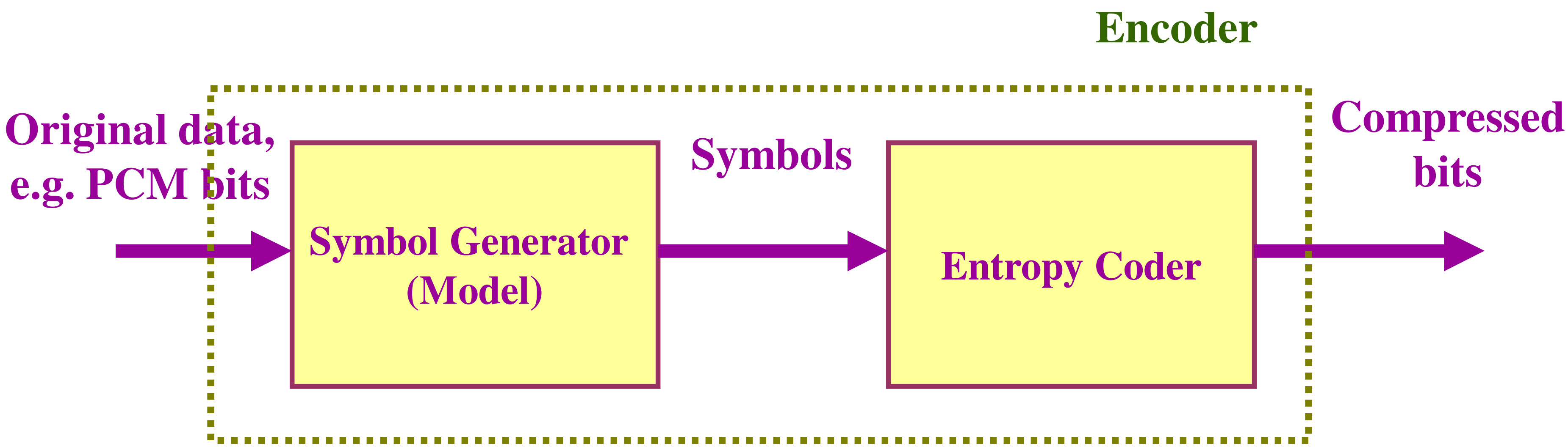
The difference between the resources requested by compressed and non-compressed formats may lead to the emergence or not of new industries, e.g., DVD, digital TV.

Digital Source Coding/Compression

Process through which a source, e.g., images, audio, video, is digitally represented considering relevant requirements such as compression efficiency, error resilience, random access, complexity, etc.

- Example 1: Maximizing the quality for the available rate
- Example 2: Minimizing the rate for a target quality

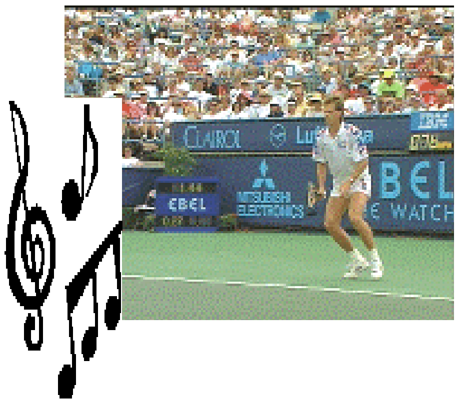
Source Coding: Original Data, Symbols and Bits



The encoder represents the original digital data (PCM) as a sequence of symbols, and later bits, using in the best way the set of available coding tools, to satisfy the relevant requirements.

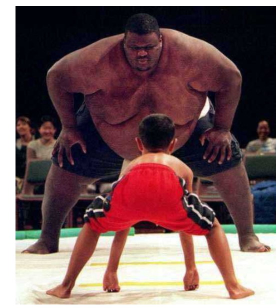
The encoder extracts from the original data 'its best' ...

Coding … and Decoding ...



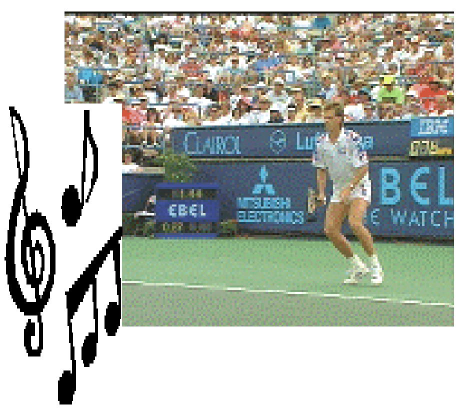
PCM Bits

Encoder



Compressed bits

Much less bits !!!

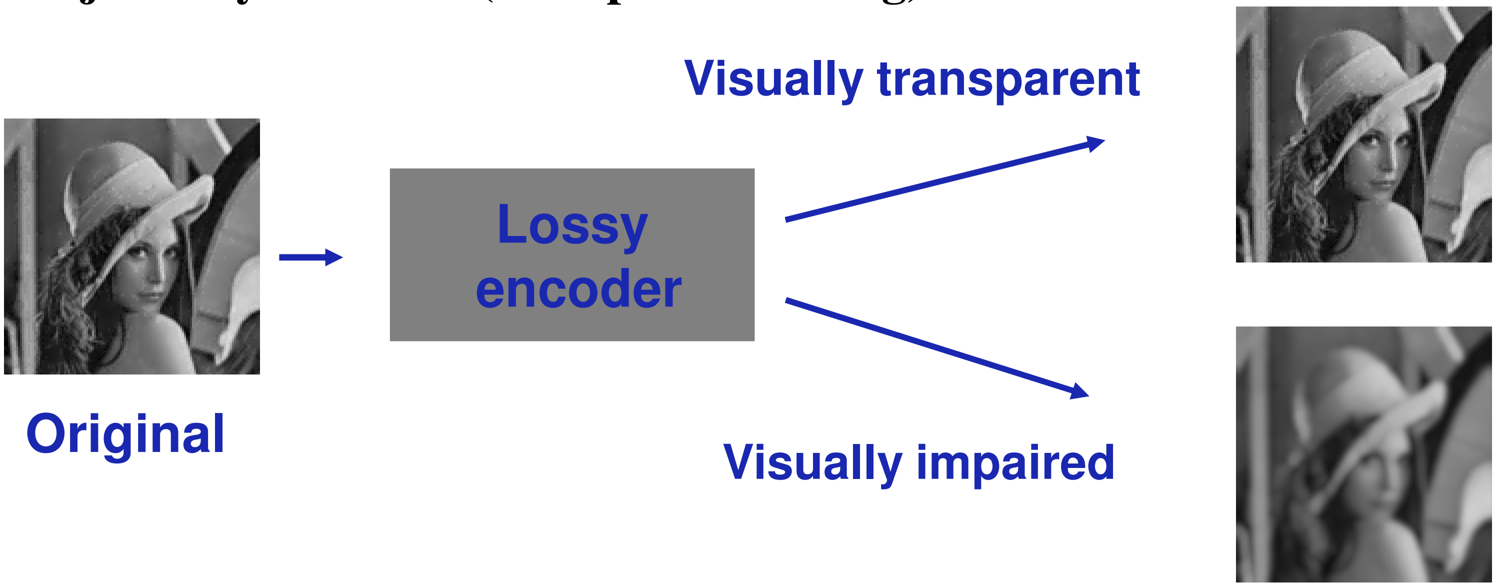


Decoder

Digital Image Coding: Main Types

- **LOSSLESS (exact) CODING** – The image is coded preserving all the information present in the digital image; this means the original and decoded images are mathematically the same.

- **LOSSY CODING** – The image is coded without preserving all the information present in the digital image; this means the original and decoder images are mathematically different although they may still be subjectively the same (transparent coding).



Where does Compression come from ?

- **REDUNDANCY – Regards the similarities, correlation and predictability of samples and symbols corresponding to the image/audio/video data.**

-> redundancy reduction does not involve any information loss this means it is a reversible process –> *lossless coding*

- **IRRELEVANCY – Regards the part of the information which is imperceptible for the visual or auditory human systems.**

-> irrelevancy reduction is an irreversible process -> *lossy coding*

Source coding exploits these two concepts: for that, it is necessary to know the source statistics and the human visual/auditory systems characteristics.

Information Theory: Entropy

Information Theory states that there is a lower limit for the average number of bits per symbol when coding *m* symbols from a source of information, each one with probability p_i. This limit is given by the source entropy obtained by:

$$H = \sum p_i \log_2 (1/p_i) \quad \text{bit/symbol}$$

- **The source entropy:**
  - **Measures the average amount of information carried by each symbol output by the source**
  - **Is a convex function of the probabilities p_i**
  - **Takes its maximum value when all symbols are the same probability (all p_i are the same)**
  - **Takes a maximum value of $log_2\, m$ bit/symbol**

Information Theory does not indicate how to obtain a code with this coding efficiency but there are methods which allow to obtain codes with an efficiency as close as desired to the entropy efficiency.

Statistical Redundancy

Statistical Redundancy of a source, *R(X)*, is defined as the difference between the maximum possible entropy of a source with the same number of symbols (*m*) and the actual source entropy

$$R(X) = \log_2 m - H(X)$$

$R(X) > 0$ implies predictability of the symbols created by the source and manifests itself in two ways:

- **Statistical dependency (memory) between the sucessive symbols of the signal**
- **Unequal probability distribution of the source symbols**

Exploiting these factors leads to a reduction of the data associated to the representation of an image, video, audio, etc.

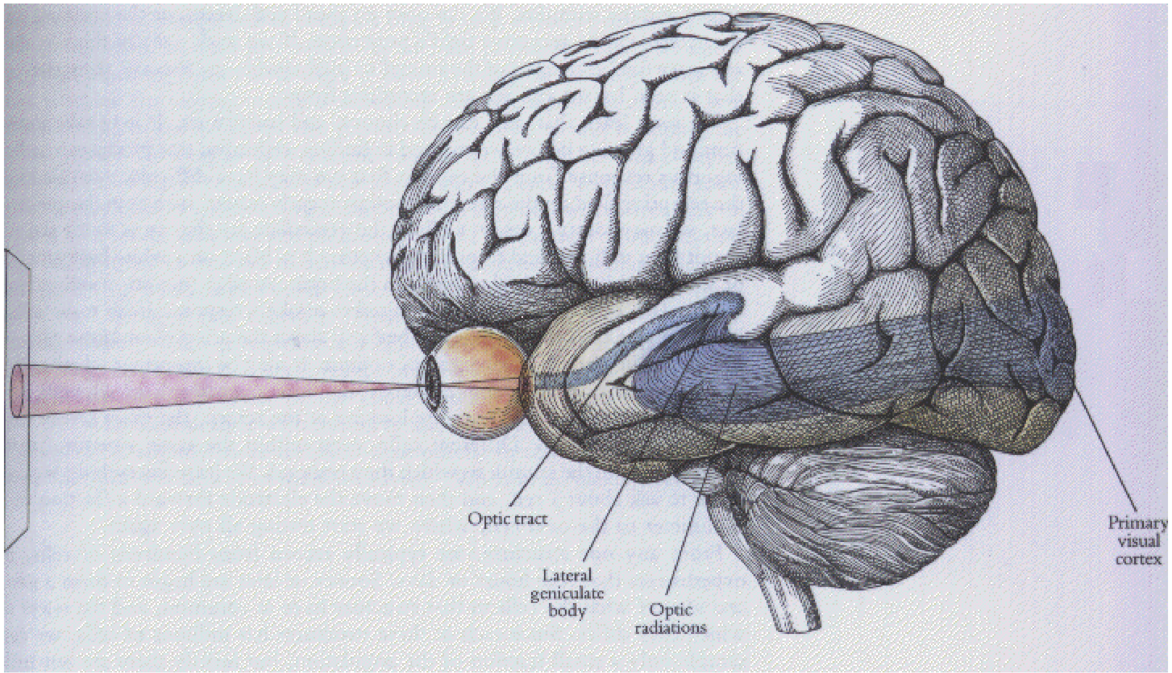
Compression Metrics

$$\text{Compression Factor} = \frac{\text{Number of bits for the original PCM image}}{\text{Number of bits for the coded image}}$$

$$\text{Bit/pixel} = \frac{\text{Number of bits for the coded image}}{\text{Number of pixels in the image (typically Y samples)}}$$

The number of pixels of an image corresponds to the number of samples of its component with the highest resolution, typically the luminance.

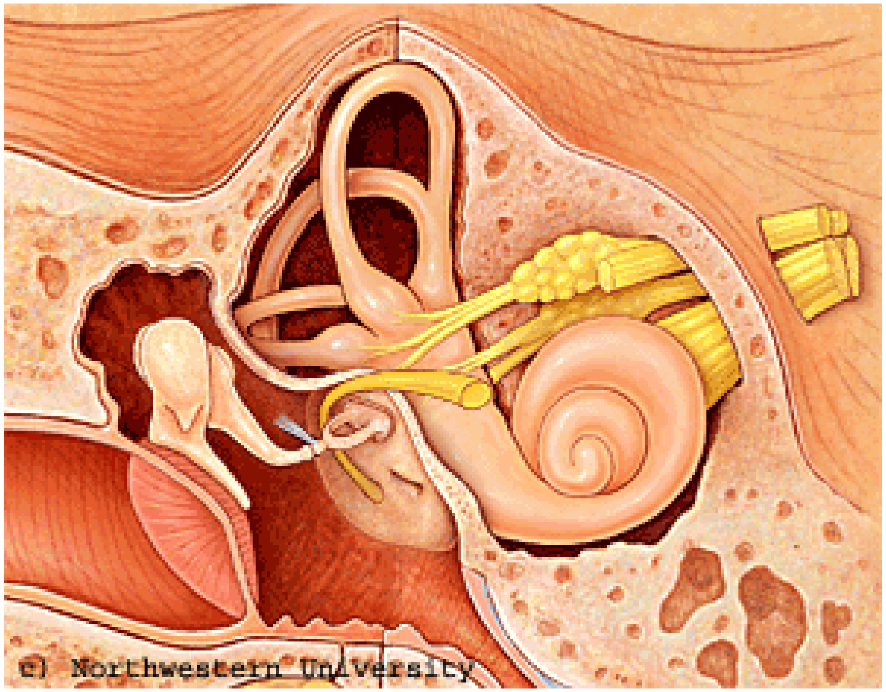
Human Visual System



It is essential to keep in mind that visual information is to be consumed by the Human Visual System !

The Human Visual System is the client that must be satisfied in terms of visual quality!

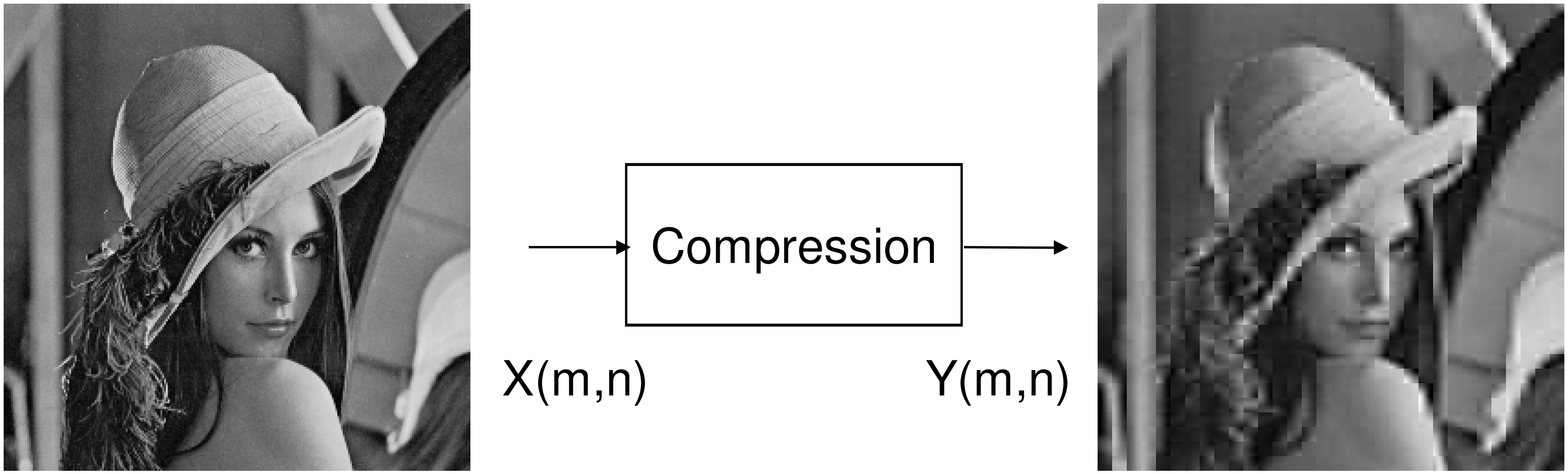
Human Auditory System



It is essential to keep in mind that audio/speech information is to be consumed by the Human Auditory System !

The Human Auditory System is the client that must be satisfied in terms of audio quality!

Quality Metrics



Subjective evaluation ⇒ e.g., scores in a 5 levels scale

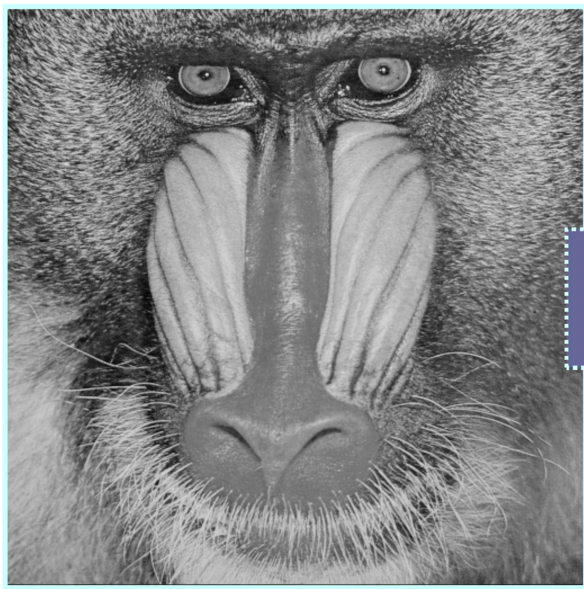
$$\text{PSNR(dB)} = 10\log_{10}\frac{255^2}{\text{MSE}}$$

Objective evaluation ⇒

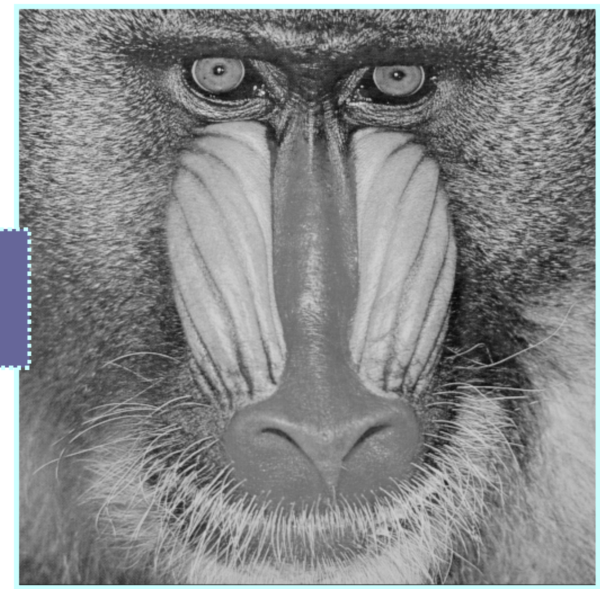
$$\text{MSE} = \frac{1}{\text{MN}}\sum_{i=1}^{M}\sum_{j=1}^{N}(y_{ij} - x_{ij})^2$$

x and y are the original and decoded data

How Does PSNR Fail …



Horizontally mirrored!



PSNR: 50.98 dB

Subjective quality: X

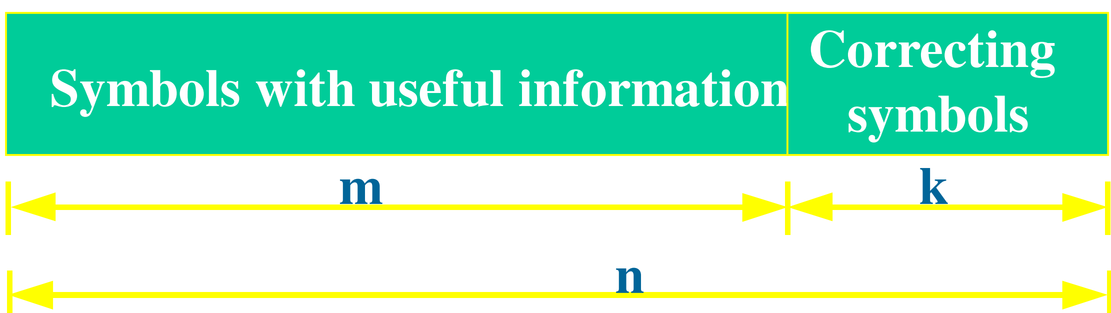
PSNR: 14.59 dB

Subjective quality: X ?

Channel Coding

Channel coding is the process applied to the bits produced by the source encoder to increase its robustness against channel or storage errors.

- **At the sender, redundancy is added to the source compressed signal in order to allow the channel decoder to detect and correct channel errors.**
- **The introduction of redundancy results in an increase of the amount of data to transmit. The selection of the channel coding solution must consider the type of channel, and thus the error characteristics, and the modulation.**



Block Codes

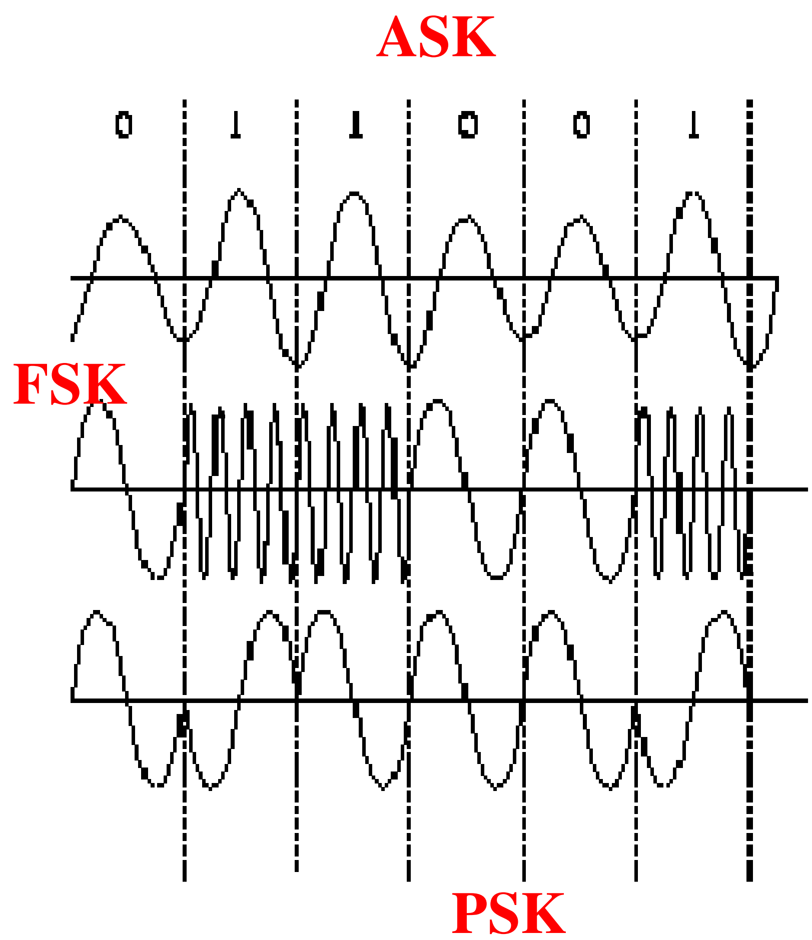
$$R = m/n = 1 - k/n$$

Digital Modulation

Modulation is the process through which one or more characteristics of a carrier (amplitude, frequency or phase) vary as a function of the modulating signal (the signal to be transmitted).

The selection of an adequate modulation is essential for the efficient usage of the bandwidth of any channel.

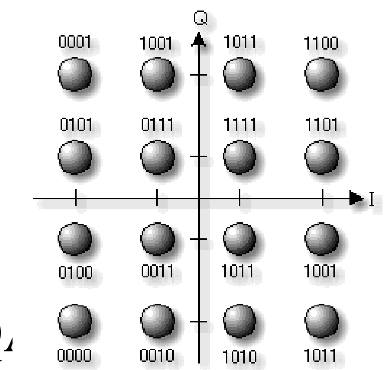
Together, (source and channel) coding and modulation determine the bandwidth necessary for the transmission of a certain signal.



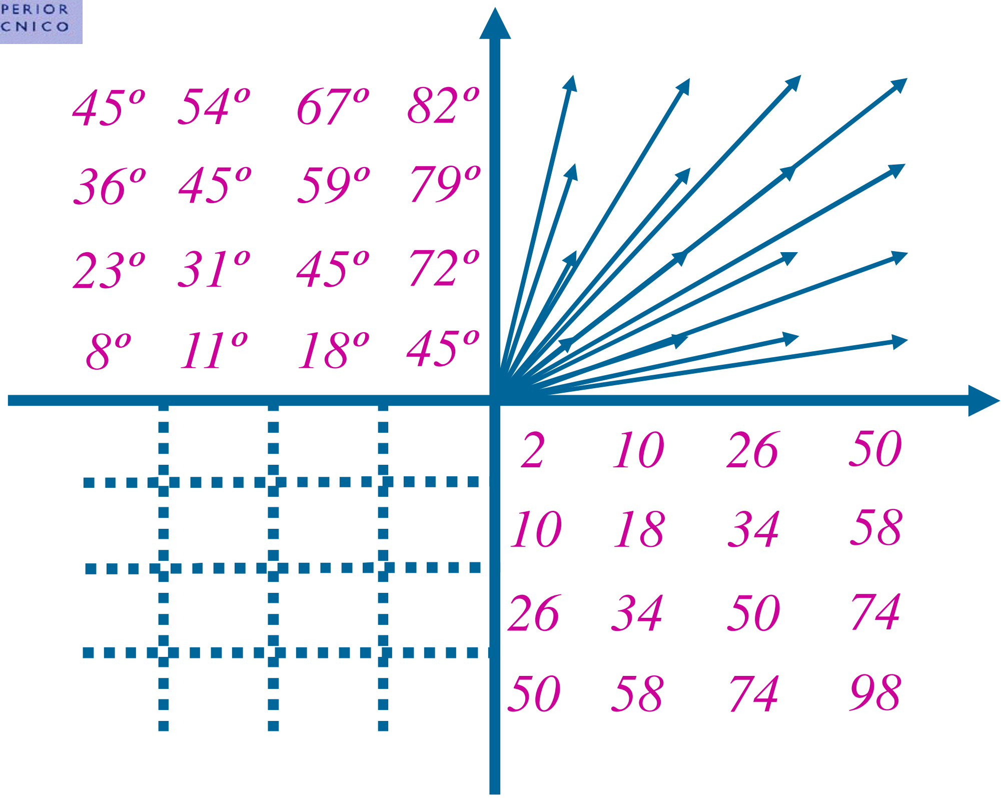
Selecting a Modulation ...

- **Factors to consider in selecting a modulation:**
  - Channel characteristics
  - Spectrum efficiency
  - Resilience to channel distortions
  - Resilience to transmitter and receiver imperfections
  - Minimization of protection requirements against interferences

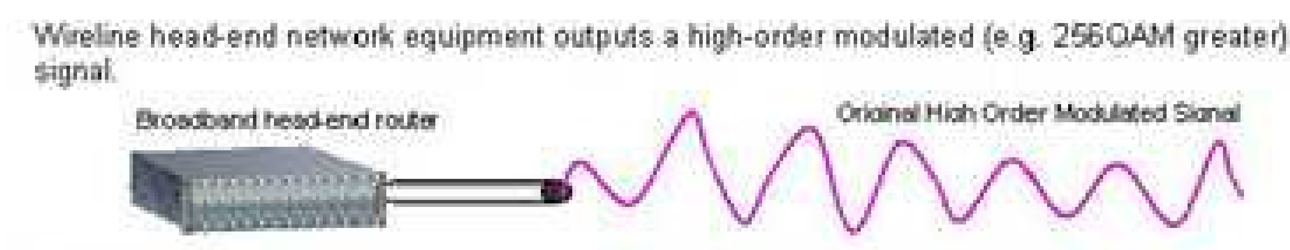
- **Basic digital modulation techniques:**
  - Amplitude modulation (ASK)
  - Frequency modulation (FSK)
  - Phase modulation (PSK)
  - Mix of phase and amplitude modulation (Q



64-QAM Modulation Constelation



For 64-QAM, only 64 modulated symbols are possible !



Digital TV: a Full Example

- ITU-R 601 Recommendation: 25 images/s with 720×576 luminance samples and 360×576 samples for each chrominance with 8 bit/sample

  $[(720\times576) + 2 \times (360 \times 576)] \times 8 \times 25 =$ **166 Mbit/s**

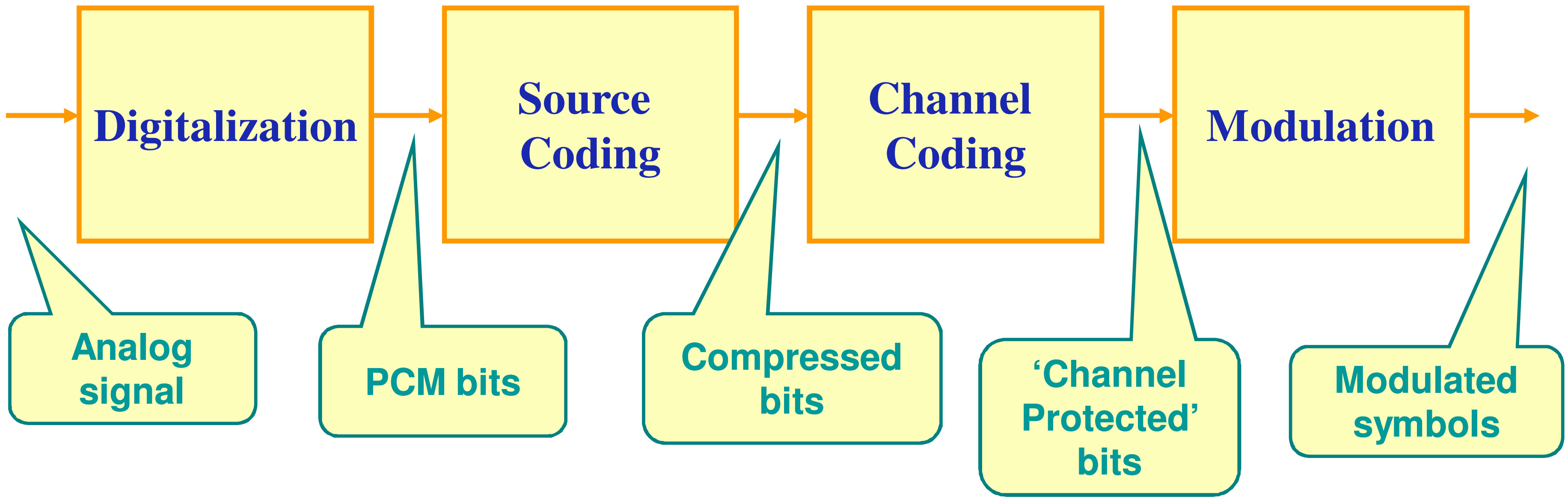
- Acceptable rate after source coding/compression, p.e. using H.264/AVC: **2 Mbit/s**

- Rate after 10% of channel coding 2 Mbit/s + 200 kbit/s = **2.2 Mbit/s**

- Bandwidth per digital TV channel, e.g. with 64-PSK or 64-QAM: 2.2 Mbit/s / $\log_2 64 \approx$ **370 kHz**

- Number of digital TV channels / analog channel: 8 MHz / 370 kHz ≈ **20 channels**

Typical Digital Transmission Chain ...



Bibliography

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- Digital Video Processing, A. Murat Tekalp, Prentice Hall, 1995