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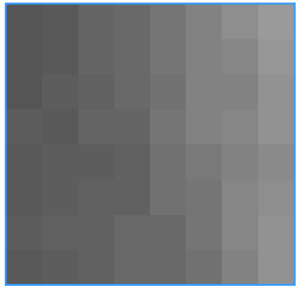
DISCRETE COSINE TRANSFORM

Laboratory session



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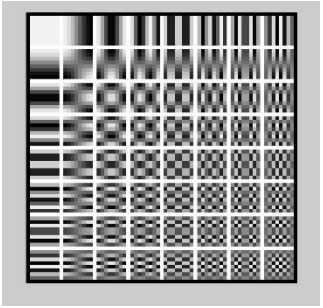
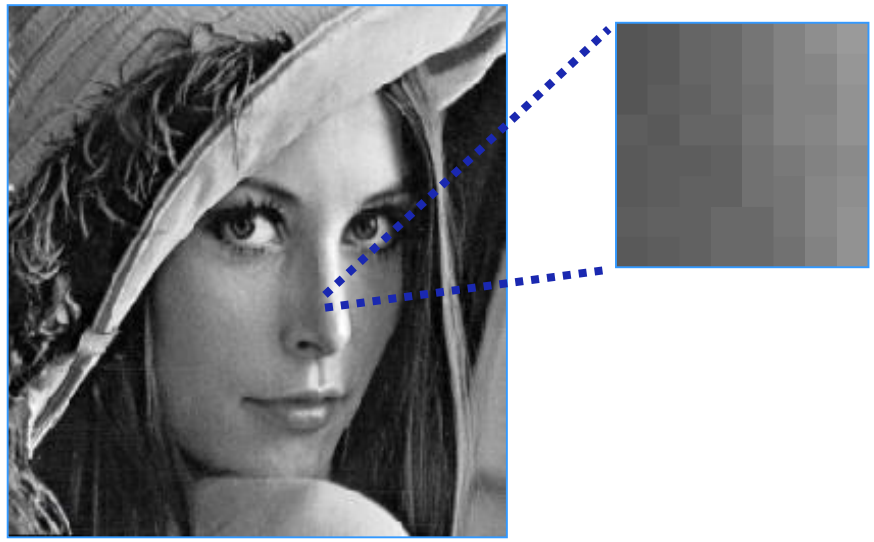


What is Transformed ? The Samples !



Transform is applied to each image component, block after block ...

Same process (in parallel) for luminance and the chrominances !

$$Y = \begin{bmatrix} 87 & 89 & 101 & 106 & 118 & 130 & 142 & 155 \\ 85 & 91 & 101 & 105 & 116 & 129 & 135 & 149 \\ 86 & 92 & 96 & 105 & 112 & 128 & 131 & 144 \\ 92 & 88 & 102 & 101 & 116 & 129 & 135 & 147 \\ 88 & 94 & 94 & 98 & 113 & 122 & 130 & 139 \\ 88 & 95 & 98 & 97 & 113 & 119 & 133 & 141 \\ 92 & 99 & 98 & 106 & 107 & 118 & 135 & 145 \\ 89 & 95 & 98 & 107 & 104 & 112 & 130 & 144 \end{bmatrix}$$

Where does Compression come from ?

- **REDUNDANCY – Regards the similarities, correlation and predictability of samples and symbols corresponding to the image/audio/video data.**

-> redundancy reduction does not involve any information loss, implying it is a reversible process –> *lossless coding*

- **IRRELEVANCY – Regards the part of the information which is imperceptible for the visual or auditory human systems.**

-> irrelevancy reduction involves removing non-redundant information, implying it is an irreversible process -> *lossy coding*

Source coding exploits these two concepts: for this, it is necessary to know the source statistics and the human visual/auditory systems characteristics.

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Exploiting the Spatial Redundancy

Discrete Cosine Transform (DCT)

The DCT is one of the several sinusoidal transforms available; its basis functions correspond to discretized sinusoisal functions.

DCT Transform coefficients

Image block

$$F(u,v) = \frac{2}{N} C(u) C(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{(2x+1)u\pi}{2N}\right] \cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

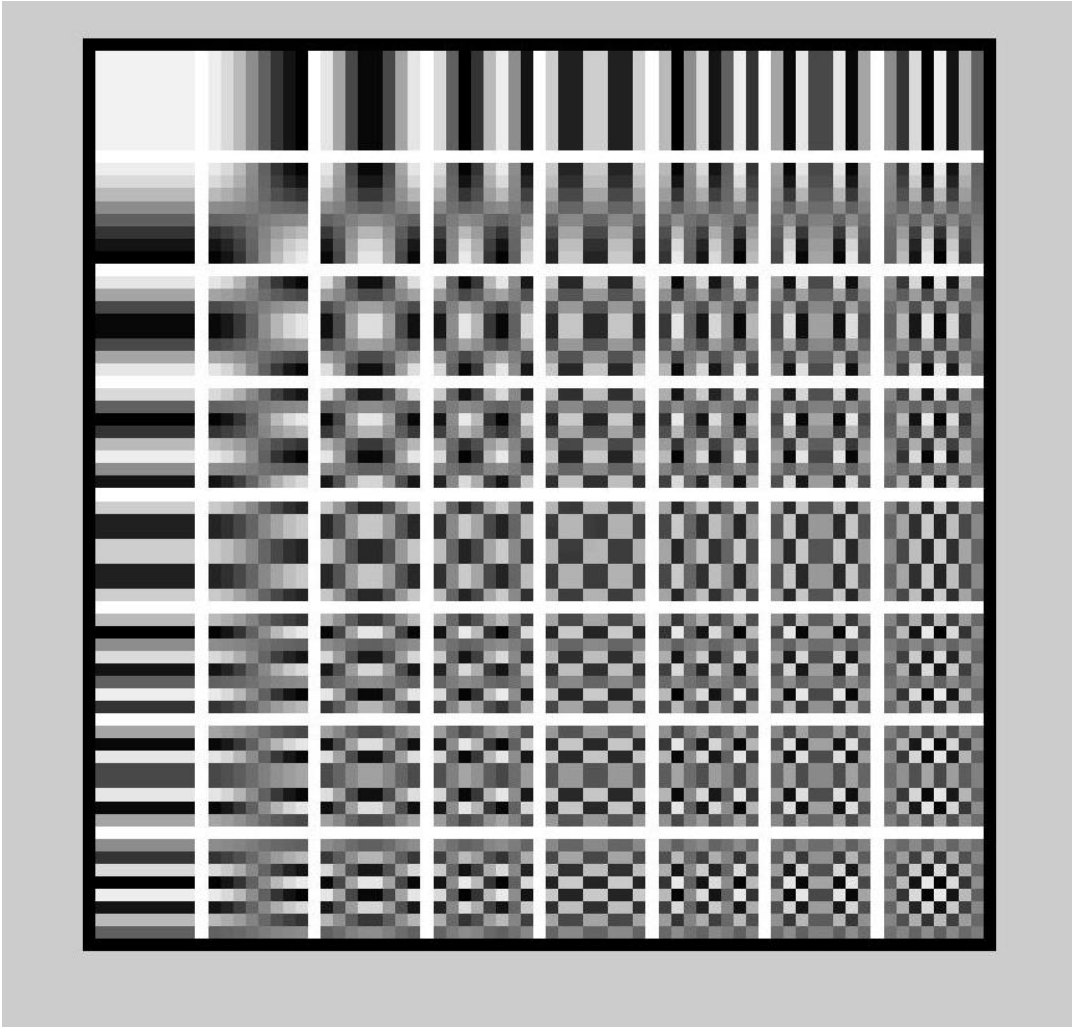
$$f(i,j) = \frac{2}{N} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} C(u) C(v) F(u,v) \cos\left[\frac{(2x+1)u\pi}{2N}\right] \cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

Image block

DCT Transform coefficients

The DCT is the most used transform for image and video coding since its performance is close to the KLT performance for highly correlated signals; moreover, there are fast implementation algorithms available.

DCT Bidimensional Basis Functions (N=8)



You see here 64 8x8 sample blocks !

All existing and future image blocks can be rather efficienctly represented with these 64 (8×8) basic images !!!

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Luminance Samples, Y =

87	89	101	106	118	130	142	155
85	91	101	105	116	129	135	149
86	92	96	105	112	128	131	144
92	88	102	101	116	129	135	147
88	94	94	98	113	122	130	139
88	95	98	97	113	119	133	141
92	99	98	106	107	118	135	145
89	95	98	107	104	112	130	144

64 PCM samples are transformed into 64 DCT coefficients !

But more perceptual compression friendly !

DCT

DCT Coefficients =

898.0000	-149.5418	26.6464	-14.0897	0.7500	-5.7540	3.5750	0.0330
12.1982	-16.5235	-7.6122	5.2187	-0.2867	-1.9909	8.4265	1.2591
5.3355	-2.6557	2.3410	-9.9277	2.4614	4.4558	-3.1945	-3.1640
1.9463	-2.7271	1.5106	2.8421	-2.1336	-2.7203	-2.7510	5.4051
0.7500	-2.0745	0.8610	0.2085	2.5000	1.8446	2.0787	2.4750
7.9536	-2.6624	2.6308	0.4010	0.4772	3.3000	1.7394	0.3942
-4.1042	-0.1650	-0.6945	0.0601	0.0628	-0.7874	-0.8410	0.3496
-3.4688	2.3804	0.1559	0.8696	0.1142	-0.5240	-3.9974	-5.6187

How Does the DCT Work ?

Spatial Domain, samples

X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X

8×8×8=512 bits

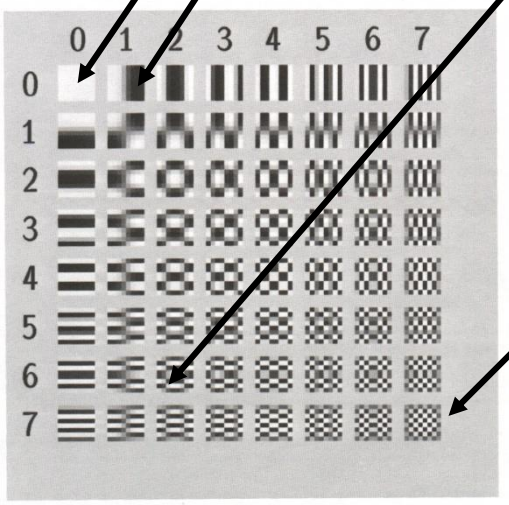
DCT

Frequency Domain, DCT coefficients

x	y		a				
C	f	d	B	c			
	H			k			
Y			i	p			
w	q	d					
	n	m					
							z

Low frequency, high energy

High frequency, low energy



Average (DC term) and Variations (AC Terms)

(a) Only DC term (PSNR = 23.66 dB)

(b) DCT without DC term (PSNR = 5.67 dB)

Building Quality, Coefficient by Coefficient …

(a) 2 coeff retained
(PSNR =21.3883)

(b) 6 coeff retained
(PSNR=22.2594)

(c) 10 coeff retained
(PSNR=24.6016)

(d) 14 coeff retained
(PSNR =25.101)

(e) 18 coeff retained
(PSNR=25.5983)

(f) 20 coeff retained
(PSNR=25.8079)

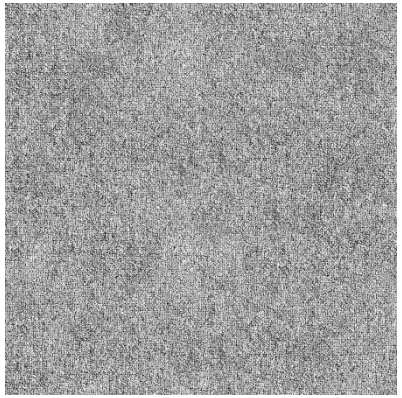
(g) 24 coeff retained
(PSNR=25.9145)

(h) 32 coeff retained
(PSNR=27.8466)

Easy/Cheap versus Though/Expensive Blocks

8×8 samples

8×8 samples



8×8 samples

All blocks above have the same price (8×8×8)=512 bits in the PCM/spatial domain because redundancy is not exploited !

In the DCT/frequency domain, simpler blocks will be cheaper and vice-versa because ‘information’ is bought with more DCT coefficients and associated rate.

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Exploiting the Perceptual Irrelevance

How Does DCT Coding Work ?

Samples (spatial domain) s_{ij}

DCT

DCT Coefficients Sij

Quantization

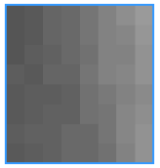
Round (S/Q)

Level for Quantized coefficients Sqij

Transmission or storage

Quantization tables Qij

≠ ≠



Dec. samples (spatial domain) r_{ij}

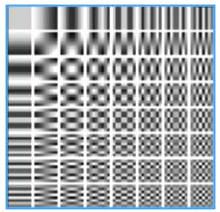
IDCT

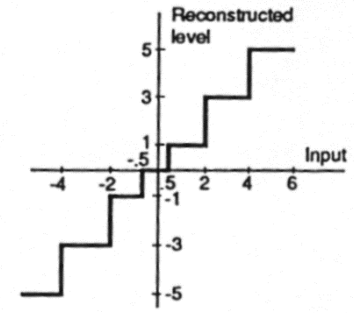
Reconstructed DCT coefficients Rij

e.g. R = Sq*Q

Level for Quantized coefficients Sqij

Inverse quantization (with quantization error)





Quantization Matrices

For transparent quality, JPEG suggests to quantize the DCT coefficients using the values for the 'minimum perceptual difference' (for each coefficient) multiplied by 2; for more compression, a multiple of them may be used.

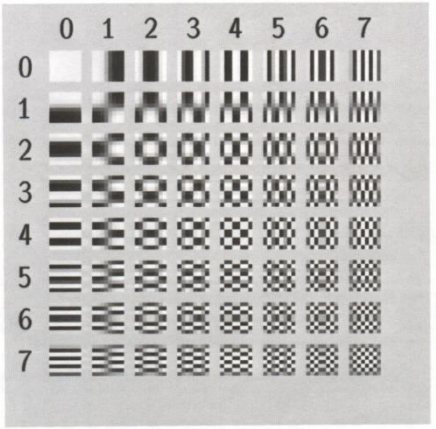
The quantization matrixes have to be always transmitted or at least signalled.

Luminance

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Chrominances

17	18	24	47	99	99	99	99
18	21	26	66	99	99	99	99
24	26	56	99	99	99	99	99
47	66	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99



Situation: Luminance and crominance with 2:1 horizontal subsampling; samples with 8 bits (*Lohscheller*)

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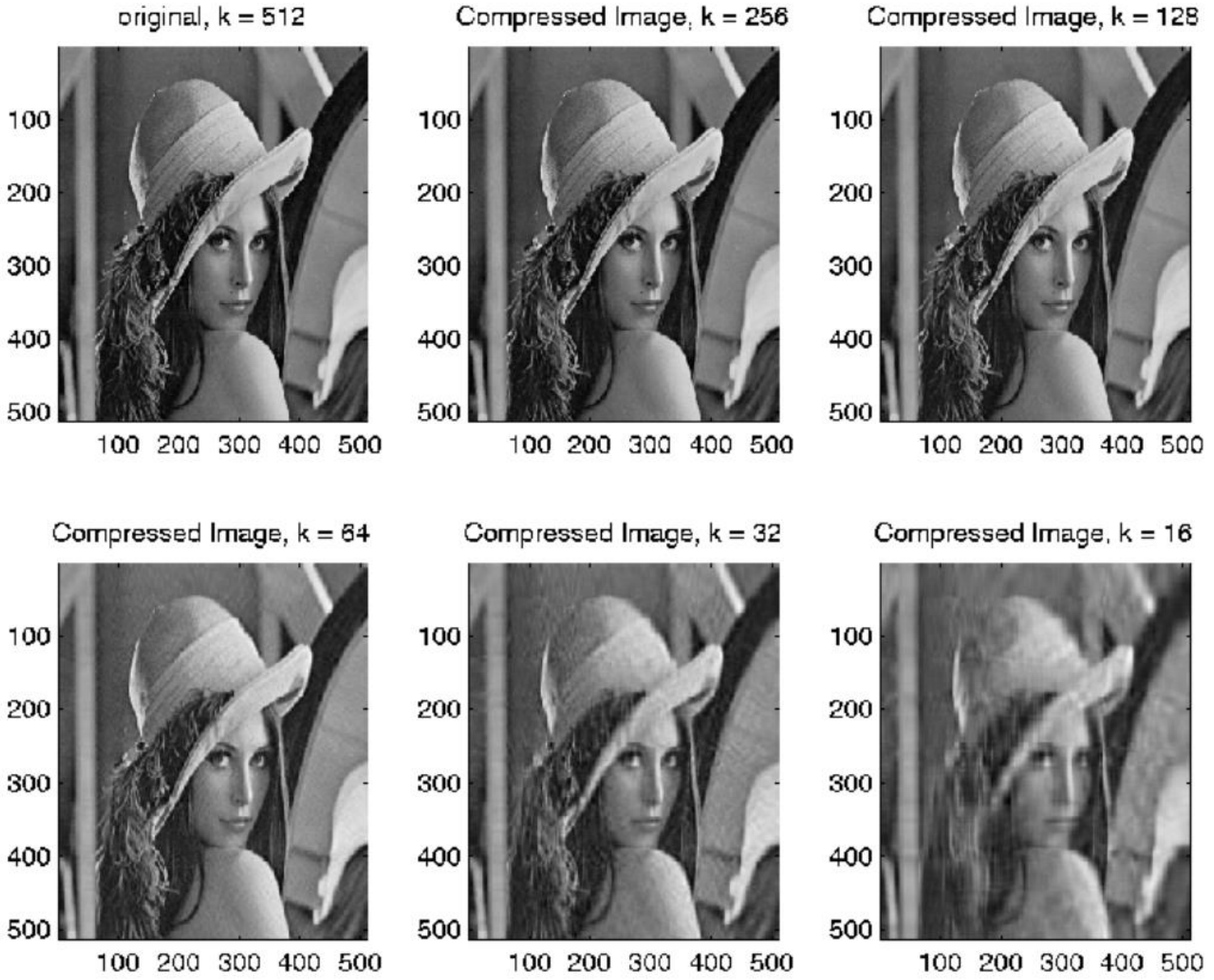
"Increasing the Quantization" …

k is the number of quantization levels



From DCT Coeffs to Quantized DCT Coeffs

898.0000	-149.5418	26.6464	-14.0897	0.7500	-5.7540	3.5750	0.0330
12.1982	-16.5235	-7.6122	5.2187	-0.2867	-1.9909	8.4265	1.2591
5.3355	-2.6557	2.3410	-9.9277	2.4614	4.4558	-3.1945	-3.1640
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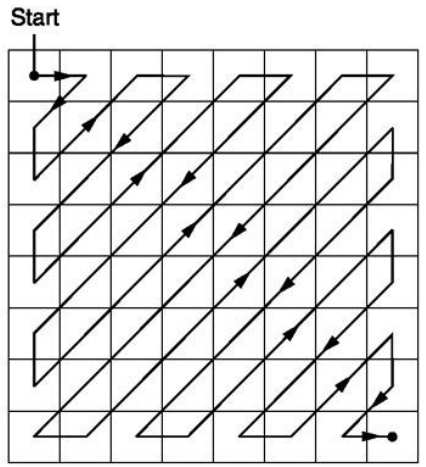
-3.4688	2.3804	0.1559	0.8696	0.1142	-0.5240	-3.9974	-5.6187

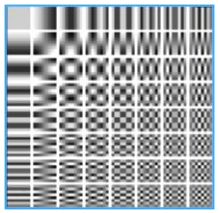
Quantizing with selected quantization matrix …

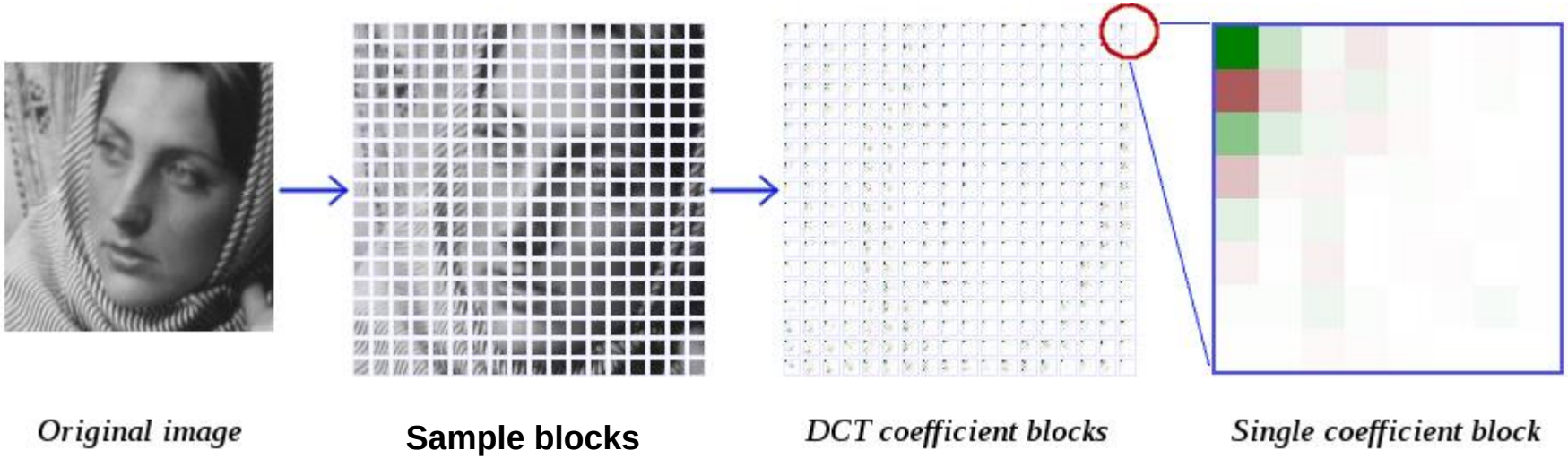
The rate is reduced, eventually at no quality cost !

56	-14	3	-1	0	0	0	0
1	-1	-1	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



The DCT+ Quantization Pipeline





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Quality Assessment

Visual (Subjective) Assessment

Full-Reference Objective Quality Assessment

Coding

x(i,j) y(i,j)

Original/reference

Decoded

Objective evaluation

x and y are the original and decoded data

$$\mathrm{PSNR(dB)} = 10\log_{10}\frac{255^2}{\mathrm{MSE}}$$

$$\mathrm{MSE} = \frac{1}{\mathrm{MN}}\sum_{i=1}^{M}\sum_{j=1}^{N}(y_{ij} - x_{ij})^2$$

$$SNR = 10\log_{10}\sum_{i=1}^{M}\sum_{j=1}^{N}x^2{}_{ij} \Big/ \sum_{i=1}^{M}\sum_{j=1}^{N}(x_{ij} - y_{ij})^2$$

There are other objective quality metrics !

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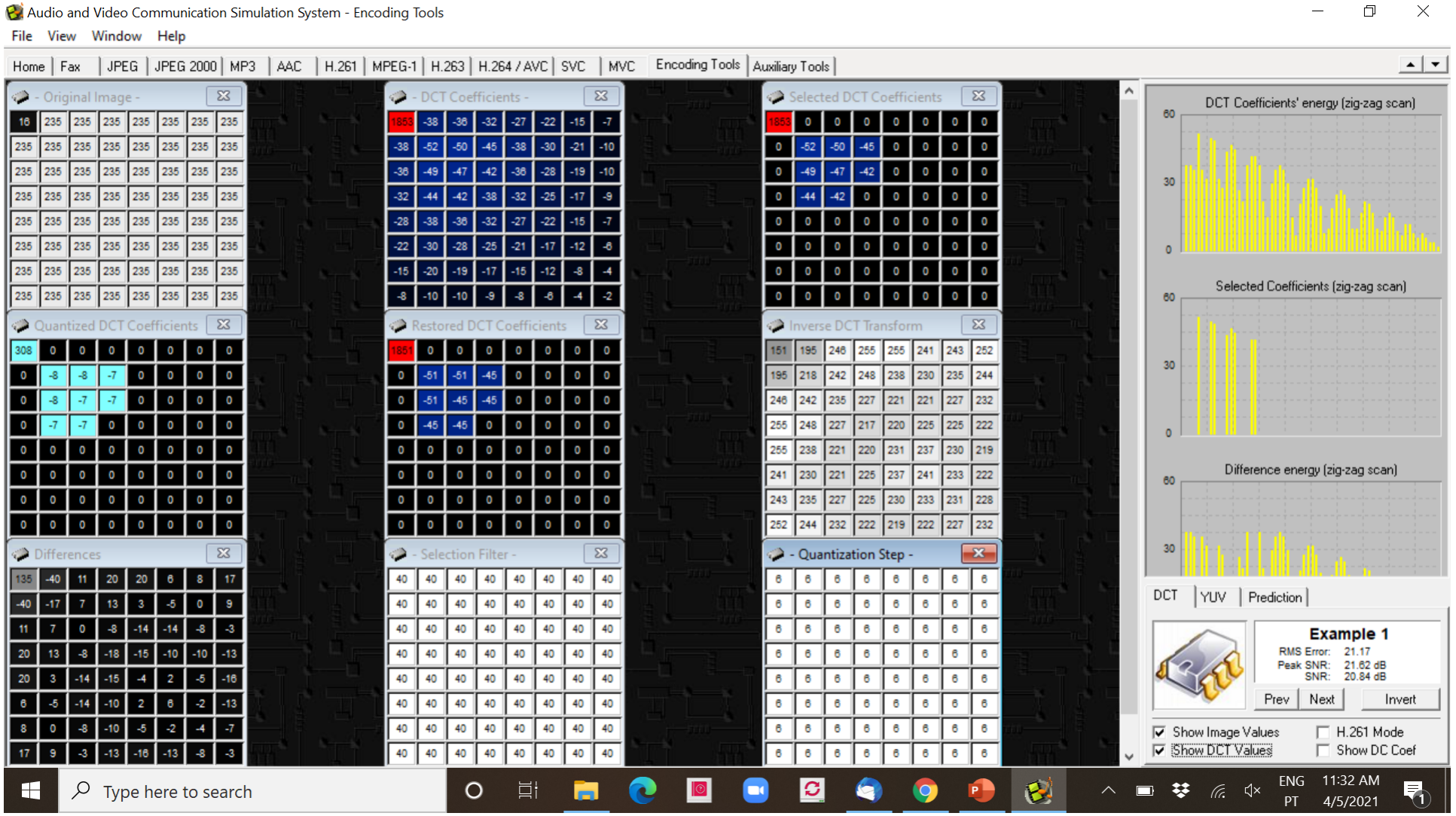

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